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Future Teachers’ Knowledge of Real numbers

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Abstract. This work is based on observations from a course at the University of Copenhagen for students who will become mathematics teachers in secondary schools. We used the anthropological theory of didactical (ATD) as our theoretical framework. Our focus is to investigate how one can develop students’ mathematical and didactical knowledge about real numbers through work with a computer algorithm that computes some zero of a given function, decimal by decimal.

Résumé. Cette présentation est fondée sur des observations faites dans le contexte d’une module de formation des professeurs de matématiques au secondaire, faite à l’Université de Copenhague. Nous avons employé la Théorie Anthropologique du Didactique (TAD) comme notre cadre théorique. Nous nous focalisons sur la possibilité de développer les connaissances mathématiques et didactiques des nombres réels par le moyen d’un travail avec une algorithme (implémenté sur Maple) qui calcule un point zéro d’une fonction donnée.

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1. Problems about real numbers in secondary school

The introduction of real numbers in secondary school is typically somewhat vague and, at crucial points, absurd. A central feature is the explanation of the meaning of irrational and real numbers. González-Martín, Giraldo and Souto (2013) investigated this case in Brazil based on the textbooks for secondary schools. They found that the set of real numbers are defined as all rational and irrational numbers. Formally, we can write this as $\mathbb{R} = \mathbb{I} \cup \mathbb{Q}$, where $\mathbb{R}$, $\mathbb{I}$ and $\mathbb{Q}$ are the sets of real, irrational, and rational numbers respectively. On the other hand, irrational numbers are characterized as real numbers which are not rational, i.e. $\mathbb{I} = \mathbb{R} \backslash \mathbb{Q}$. While rational numbers are given an explicit definition in terms of fractions, we observe that the meaning of real numbers remain entirely in the dark, even with the occasional complement that they can all be written as "infinite decimal numbers".

Another problem, further explored by Zazkis and Sirotic (2004), is a "missing link" between the fraction representation of rational numbers and their decimal representation, in particular when it comes to characterizing rationality.

2. Theoretical framework and research questions

One possible way to approach the above two problems, at least for teachers, is to formalize the idea that real numbers can be defined as infinite decimals. Considering the widespread use of digital technology, we observe that numbers are habitually represented as (finite) decimals by calculators or computers. It is natural to integrate such tools in efforts to support mathematics students' knowledge of real numbers.

Our work follows up on the research by Barquero et al (to appear). In particular we investigate the second gap of Klein's "double discontinuity" (Klein, 2016).

Chevallard (2019) denoted the relation between a position p within the institution I, and the knowledge object o, by $R_I(p, o)$. In ATD, a knowledge object is modeled as a praxeology. In secondary school, the praxeologies related to real numbers which should be taught/learnt (denoted by $o_{\mathbb{R}}$) focus more on practice blocks. This may lead teachers to ignore the theory blocks. For example, students or teachers care more about the correction of the calculation between real numbers. On the contrary, the knowledge of real numbers (denoted by $\omega_{\mathbb{R}}$) which should be taught/learnt in university pays more attention to theory blocks. The teaching/learning turns to study the properties of real number like "completeness" rather than the calculation between real numbers. Therefore, in a university course aimed at improving future teachers' grasp of real numbers in view of teaching, a possible strategy is to draw on theory blocks from $\omega_{\mathbb{R}}$ to explore and explain practice blocks from $o_{\mathbb{R}}$, which may often require adding "intermediate" elements of theory and practice. Thus, such a course will involve a kind of "closure" or "convex hull", denoted by $\overline{o_{\mathbb{R}} \cup \omega_{\mathbb{R}}}$, and aims to establish a new relationship $R_U\left(\sigma_{ft}, \overline{o_{\mathbb{R}} \cup \omega_{\mathbb{R}}}\right)$ for future teachers σ_{ft}. Our first research question is about the content that could be contained in $\overline{o_{\mathbb{R}} \cup \omega_{\mathbb{R}}}$: RQ1. How could the idea of 'infinite decimal' be related to university mathematics and taught to future secondary school teachers?

In particular, how could students select suitable theoretical elements from $\omega_{\mathbb{R}}$ as they reconsider tasks and objects habitually found in $o_{\mathbb{R}}$? As a way to bridge Klein's second gap for this particular case, we thus consider to introduce the following transition :

$$R_U\left(\sigma_{ft}, \omega_{\mathbb{R}}\right) \to R_U\left(\sigma_{ft}, \overline{o_{\mathbb{R}} \cup \omega_{\mathbb{R}}}\right) \qquad (1)$$

In Denmark, Computer Algebra Systems (CAS) like *Maple*, *TI Nspire* and *Geogebra* are commonly used at secondary schools. At university, such tools only appear in introductory Calculus and Linear Algebra courses. Therefore, we added a more specific question concerning the transition (1): RQ2: How can working with a given computer algorithm support university students' use of (university level) technical and theoretical knowledge to address secondary school level questions related to the infinite decimal model of real numbers? What new mathematical and didactical knowledge on decimal representations of real numbers can such work enable students to develop?

3. Context

In order to investigate these two research questions, we conducted an experiment in an optional course called UvMat at the University of Copenhagen (Denmark) for future mathematics teachers. In this course students learnt how to think high school mathematics from a university level. There are around 20-30 students every year in this course and most of them completed around 2 years of study of mathematics before attending. Throughout the course, students attended a lecture and an exercise class following the textbook (Sultan & Artzt, 2018), and they also do an assignment in groups every week (totally 7 weeks). The fourth course week is based on the second part of chapter 8 of the textbook, dealing with real numbers. A main focus is to formalize and explore, from a university perspective that includes Cauchy's definition of real numbers, the representation of real numbers as (infinite) decimals, in the form: $\pm N.c_1c_2c_3 \ldots = \pm N + \sum_{i=1}^{\infty} c_i \cdot 10^i$. A main result is that all real numbers can be uniquely represented in this form if we exclude the case where c_i becomes eventually 0.The details of the textbook can be an example of how to answer the RQ1.

We designed the weekly group assignment for this 4th week of the course (the whole assignment is included as an appendix). This assignment is an example of a long study and research line (SRL). Some examples of short SRL are introduced by Winsløw (in preparation). Two parts are contained in this assignment. The first part relates to students' basic understanding of a given computer routine. This routine can be used to find the first 10 digits of $\sqrt{2}$ on *Maple* one by one, based (theoretically) on the intermediate value theorem. We denote $x(n)$ as the first n digits of decimal x, for example if $x = N.c_1c_2c_3 \ldots$, then $x(2) = N.c_1c_2$. This part entails question b), c), d) and e). We use this part to investigate how students present and explain real numbers as decimals, drawing on *Maple*. The second part is the question f), which is used to investigate how students understand the addition of infinite decimals. This part is not independent from the first part. In this part, students are expected to compare the data they obtained from the first part and give a conclusion from their observations.

After we reviewed students' answers, we interviewed 5 students from different groups about their assignment answers through voluntary participation. Students were asked to explain their answers to the visualizations they made for the question c) and their conclusions from question f). Their answers and explanations to these two questions are used to answer the RQ2.

4. Results

We received 8 groups' assignment answers. In the first part of the assignment, students were asked to do some practices like proving the limit, making visualizations and find an appropriate polynomial. After our analysis, we found students prefer to use informal explanations (i.e. without a complete mathematical proof) based on what they learnt in secondary school. When they were facing what they perceived as a secondary school task, most of the students acted as secondary school students rather than preservice students σ_{ft} at a university. They believed it is enough to use only the methods they learned in secondary school.

The visualizations made by students is our main target (question c)) in the first part. This is a second way the students are asked to explain the given routine in the assignment. In this question, students were also required to use *Maple* by themselves. Therefore, except their knowledge to understand the routine, they also need to have the competence to operate *Maple*. Combined with students' explanations from interviews, we found that most students focused on the results produced by the routine, even though they gave some different explanations for these results. For example, one group considered about the speed of change, *"... points are getting closer much faster in the end than in the beginning…"*. From the interviews, we also know that what was difficult for them was not how to understand or explain the routine but to overcome certain lacks of knowledge of Maple. They spent a lot of time finding a way to make the figures as they planned.

Question f) is a question to examine students' theoretical knowledge on infinite decimals. As we expected, the groups who used CAS to compare the data they obtained from question b) to e) presented more reasonable answers. These students used a visualized way to make the answers more convincing and attractive.

As a high school teacher in Denmark they will both supervise pupils' use of CAS in relation to real numbers and functions, and use CAS facilities to express knowledge in a suitable way for students, which might usefully include non-trivial visual representations produced by CAS. On students' answer sheets, they gave a lot of explanation after they post the results from CAS. For example, in question c), all groups used their own way to clarify how the figures they made can explain the routine. When the teacher is teaching, the necessary explanation is an important step to help students understand the knowledge. Therefore, one could argue that assignments may support the development of both mathematical and didactical knowledge of students, but the extent to which that happens in practice needs further scrutiny.

References

Barquero, B. and Winsløw, C. (to appear). Preservice secondary school teachers revisiting real numbers: a striking instance of Klein's second discontinuity. Manuscript, accepted for forthcoming Springer book.

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: an attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, 12, 71–114.

González-Martín, A. S., Giraldo, V., & Souto, A. M. (2013). The introduction of real numbers in secondary education: an institutional analysis of textbooks. *Research in Mathematics Education*, 15(3), 230-248.

Klein, F. (2016). *Elementary Mathematics from a Higher Standpoint: Volume I: Arithmetic, Algebra, Analysis* (G. Schubring, Trans.). Berlin, Germany: Springer. (Original work published 1908)

Sultan, A. & Artzt, A. F. (2018). *The Mathematics That Every Secondary School Math Teacher Needs to Know* (2nd ed.). *Routledge*.

Zazkis, R. & Sirotic, N. (2004). Making sense of irrational numbers: Focusing on representation. In M. Høines & A. B. Fugelstaed (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol4; pp. 498-504). Bergen, Norway: PME

Winsløw, C. (in preparation). Mathematical knowledge for the profession of upper secondary mathematics teaching.

Appendix

Week Assignment 4

For $n \in \mathbb{N}$ we define $\mathbb{D}_n = \{10^{-n}x : x \in \mathbb{Z}\}$, and we define $\mathbb{D} = \cup_{n\in\mathbb{N}}\mathbb{D}_n$. Also define $\mathbb{D}_\infty$ to be the set of formal expressions $\pm N.c_1c_2...$ where $N \in \mathbb{N} \cup \{0\}$ and $c_k \in \{0, 1, ..., 9\}$ for all $k \in \mathbb{N}$, and finally let $\mathbb{D}_0$ be the set of formal expressions $\pm N.c_1c_2...c_k000...$ where $N \in \mathbb{N} \cup \{0\}$, $k \in \mathbb{N}$ and $c_1, ..., c_k \in \{0, 1, ..., 9\}$.

a) Prove that there exists bijections $\varphi : \mathbb{D}_0 \to \mathbb{D}$ and $\psi : \mathbb{D}_\infty \setminus \mathbb{D}_0 \to \mathbb{R}$, but that no bijection exists between $\mathbb{R}$ and $\mathbb{D}$.

b) Consider the following routine in Maple (try it out!):

```
K := 1 :;
for i from 0 to 10 do
      for j from 0 to 9 do
           if (K + j * 10^(-i))^2-2 <= 0 then
                p := K + j*10^(-i);
           end if ;
end do;
K := p :;
print(x(i) = evalf(p, i + 1));
end do :;
```

Explain what the routine does, why $x(n) \in \mathbb{D}_n$, and why $x(n) \to \sqrt{2}$.

c) Use Maple to produce a visual explanation of how the routine from b) works.

d) Explain how a similar routine can be made for any continuous function f, to find a zero between $a \in \mathbb{Z}$ and $a + 1$, when $f(a)f(a + 1) < 0$. How does the intermediate value theorem come into play? How can you use this idea to approximate $\sqrt{3}$ by numbers from $\mathbb{D}$?

e) Find a polynomial p such that $p(\sqrt{2} + \sqrt{3}) = 0$, and use the idea from d) to approximate $\sqrt{2} + \sqrt{3}$ by numbers from $\mathbb{D}$.

f) Investigate what the results from b), d) and e) tell you about addition on $\mathbb{D}_\infty \setminus \mathbb{D}_0$.

The role of algebraic models and theory in Danish lower secondary school.

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Abstract. In this paper, we address the external didactical transposition of algebra in Danish lower secondary school. The theory of praxeology and especially the construction of a praxeological reference model enable us to analyse what elementary algebra praxeologies are currently to be taught in Danish lower secondary school according to curriculum, textbook material, and the written examination at the end of grade 9. To exemplify the set of conditions and constraints that affect the role of algebraic modelling and level of theory in praxeology we will use "algebra models in geometry" as a case.

Résumé. Dans cet article nous considérons la transposition externe de l'algèbre tel qu'il se fait au niveau du collège au Danemark. La théorie des praxéologies et en particulier la construction d'un modèle praxéologique de référence nous permet d'analyser les praxéologies algébriques couramment supposé d'être enseignées à ce niveau, suivant le programme, les manuels et l'examen écrite situé à la fin de la neuvième année de scolarisation. Afin d'exemplifier l'ensemble de conditions et de contraintes pour le rôle de modélisation algébrique au niveau théoriques, nous étudions le cas de « modèles algébriques en géométrie ».

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Editorial, año

1. Introduction and research question

In the written examination after lower secondary school, the majority of Danish students struggle with setting up an expression for the area and perimeter of a polygon with only vertical and horizontal sides, when given symbols for the relevant lengths, Figure 3. The lack of knowledge on how to use algebra as a modelling tool is a central issue in the current debate on the "algebra problem" in Danish lower secondary school (Østergaard, 2021). The generally modest algebra skills of students become visible not only at the written exam after lower secondary school. In secondary school, basic algebra is a crucial bridge between arithmetic and more advanced subjects involving functions and analytic geometry.

According to Strømskag and Chevallard (to appear) the problem is not only a Danish one: also in France and Norway, school algebra has become a set of formal exercises, rather than a modelling tool; they argue for "an imperative revitalization of the elementary algebra curriculum" (Strømskag & Chevallard, 2022, p. 1).

In Denmark, the so-called "common goals" for mathematics constitutes the official directives for primary and lower secondary school. The overall goal for algebra after lower secondary school (grade 9), is that "the student can apply real numbers and algebraic expressions in mathematical investigations" (Danish Ministry of Education, 2019). However, teachers base their teaching on textbooks and national exams, which are often more modest in their demands when it comes to algebra.

Danish textbooks are crafted by "leading" math teachers, based on personal didactical ideas and experiences. The teachers also find some directions in the exercises appearing in the national exam after grade 9. To get insights in the set of conditions and constraints that affect the knowledge to be taught, we will look at the external didactic transposition (Bosch, M., Hausberger, T., Hochmuth, R., Kondratieva, M., & Winsløw, C., 2021), and explore the research question: *What is the role of algebraic models and theory in Danish lower secondary school?*

2. Theoretical framework and methodology

As a teacher educator, textbook writer, and Ph.D. student, I am formed by the set of institutions I am acting in. However, the use of the

Anthropological Theory of the Didactic as theoretical framework for my doctoral thesis gives me the opportunity as a researcher to detach myself from any specific institutional viewpoint (Bosch, 2015). To distance from common-sense models used within understand the institutions, we use *modelling* in the ATD sense as a didactic tool to structure and integrate modeling processes in a more general epistemological model of institutional mathematical activities (Garcia, F. J., Gascón, J., Ruiz Higueras, L. & Bosch, M., 2006).

The overall aim of the doctoral project is to investigate the transition from arithmetic to algebra and to explore if and how research-based teaching materials can support and direct teachers' efforts to teach basic algebra. In this first part of the study, we study only the external transposition to get insight into the set of conditions and constraints that affect the knowledge to be taught (Bosch et al., 2021).

To investigate the role of algebraic models and theory in Danish lower secondary school, we build a praxeological reference model (PRM) based on curriculum, textbook material, and written examination. The praxis is formed by type of tasks and by the techniques used to solve them, and logos consisting of technology and theory (Barbé, Bosch, Espinoza & Gascón, 2005). The explicit construction of a PRM will enable us to analyse what arithmetic and algebraic praxeologies are currently to be taught in Danish lower secondary school. The PRM includes themes such as and algebraic models in geometry. We will consider this theme as a case example and refer to the more comprehensive PRM (not presented here) by type of tasks T_i and corresponding techniques τ_i.

3. Algebraization in geometry

For the analyses one of the most common textbook material Kontext+ grade 5 to 9 (Lindhardt, Thomsen, Johnsen & Hansen, 2021) and the national 2019 written paper and pencil exam (Prøvebanken, 2021), are used. In the analyses two specific types of "algebra models in geometry" appear (among others). The first type of tasks is "Determine the perimeter of the polygon" denoted T_{27}, where the technique τ_{27}: Add the side lengths of the polygon, will solve this type of tasks. The second type of tasks is T_{28}: Determine the area of a polygon with all sides being either parallel or orthogonal, with the corresponding technique, τ_{28}: Calculate the area by using the formula of rectangle area together with the additive principle (the area of a disjoint union of polygons is the sum of the area of those polygons). At the level of algebraic theory, the distributive law appears.

We now analyze an example of each of this type of tasks and give an example of the theoretical approach. To analyze the examples we distinguish between "arithmetical rules" and "algebraic formulas" and the essential notion of parameters in line with Strømskag and Chevallard (2022).

3.1. Geometric multiplication of binomials

The first example is from Kontext+8 a grade 8 textbook material.

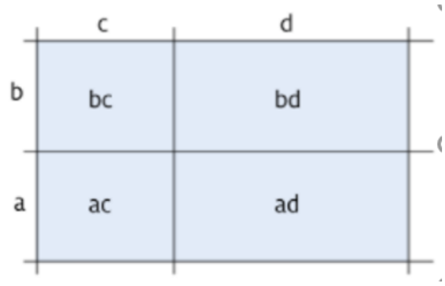

Figure 1. Floor with four rooms (Lindhardt et.al. 2021, p.87)

a. Show from the drawing, why

$$(a + b) \cdot (c + d) = ac + ad + bc + bd.$$

b. How large is the area of the four rooms if

$a = 5, b = 25, c = 10\ and\ d = 40$?

To answer question a., we can τ_{28} with the given subdivision, and no subdivision. For the latter approach, the side lengths to be used are $l = a + b$ and $b = c + d$. Then we get that the area is $A = l \cdot b = (a + b) \cdot (c + d)$. To express the area of the four small rectangles we use the formula of the area of a rectangle for each of the small rectangles and sum these up. It means that can deduce the formula from algebraic model of area, with no need to manipulate the algebraic expression.

In question b. we must interpret what is being described as rooms to be the small rectangles. Then we can use the algebraic model of area of a rectangle, and by inserting the definite numerical values we get the numeric areas. This is a technique taught in grade 5, if following the textbook cited above.

3.2. Distributive law

The distributive law is the crucial links between addition and multiplication (and a field axiom) which forms part of the level of theory of both arithmetic and algebra in the PRM. In the textbook the point is made that "you can use geometric figures to model arithmetic rules by letters". In fact, what the example in Figure 2 does is to deduce an algebraic law by using τ_{28} on a particular geometric figure.

Geometrisk algebra

Man kan bruge figurer til at vise regneregler med bogstaver.
Rektanglets sider er c og (a + b).
Arealet kan derfor skrives som c · (a + b).

Arealet kan også skrives, som summen af de to dele
af rektanglet a · c + b · c.

Altså c · (a + b) = a · c + c · b.

Figure 2. Geometric algebra (Lindhardt et.al. 2021, p.95)

Specifically, τ_{28} tells us that the area of the big rectangle is $a \cdot c + b \cdot c$, and it is also $(a + b) \cdot c$. The generality and variations of the distributive law stays implicit, and the students are not introduced to "distributivity" as an assumption or axiom in algebra. It is not visible for the student that the example is special (for instance, assumes that $a, b > 0$).

3.3. Perimeter and area of an irregular octagon

Our last example is from the national written paper and pencil examination after grade 9 (Prøvebanken, 2019).

Figure 3 shows an octagon with marked sides of length a and b.

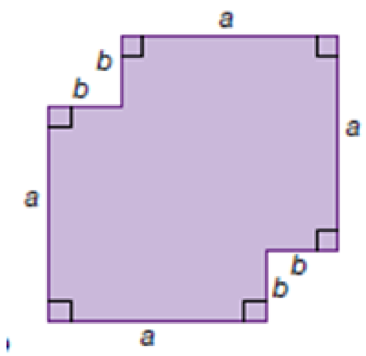

Figure 3. Irregular octagon

c. The perimeter of the octagon is ______________
d. The area of the octagon is _______________

Here, c. is of the type T_{27}, and τ_{27} yields that the circumference is $a + b + b + a + a + b + b + a$; then, another algebraic technique (τ_{15}: collect equal terms) can be used to get the final result 4a+4b.

Question d. is again of type T_{28}. Here must model the area, by dividing the octagon into rectangles, which can be done in several different ways, resulting in expressions such as $(a + b)^2 - 2b^2$ or $a^2 + ab + b(a - b)$, which of course can both be reduced to $a^2 - b^2 + 2ab$ by τ_{15}.

4. Discussion and conclusion

In the Danish curriculum for grades 6 to 9, the aim of the topic "Formulas and algebraic expressions" phase one is that "the student can describe connections between simple algebraic expressions and geometric representations" (Danish Ministry of Education, 2019).

The previous textbook examples provide examples of the use of algebraic models in geometry. Types of tasks in the theme "Algebra models in geometry" are mostly formal exercises solved by inserting values in known or given formulae. The level of theory is implicit and algebraic manipulation occurs rarely (the examples above are thus somewhat special). When algebra turns into a modelling tool, it is often in relation to simple geometric situations. And even though such modelling exercises do occur in the textbook material, the results from the written examination, where only 9.6% could answer questions c. and d. (above) properly, suggest that algebraic models and theory are not really a part of the students' topos (Chevallard, 2019).

References

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic Restrictions on the Teacher's Practice: The case of limits of Functions in Spanish High Schools. *Educational Studies in Mathematics*, 59, 235–268. https://doi.org/10.1007/s10649-005-5889-z

Bosch, M. (2015). Doing research within the anthropological theory of the didactic: the case of school algebra. In *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 51–

69). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_4

Bosch, M., Hausberger, T., Hochmuth, R., Kondratieva, M., & Winsløw, C. (2021). External didactic transposition in undergraduate mathematics. International Journal of Research in Undergraduate Mathematical Education, 7(1), 140-162. https://doi.org/10.1007/s40753-020-00132-7

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima journal of mathematics education*, *12*, 71–114.

Danish Ministry of Education (2019). Fælles mål matematik. Danish Ministry of Education. Located 10.12.2020 at https://emu.dk/grundskole/matematik/faghaefte-faelles-maal-laeseplan-og-vejledning

Garcia, F. J., Gascón, J., Ruiz Higueras, L. & Bosch, M. (2006) Mathematical modelling as a tool for the connection of school mathematics. ZDM, 2006, Vol. 38(3), pp. 226-246. http://doi.org/10.1007/BF02652807

Lindhardt, B., Thomsen, H., Johnsen, L. B. & Hansen J. (2021). KonteXt+7, i-bog. Alinea. Located 01.03.2021 at https://kontextplus.alinea.dk/course/QRAT-kontext-7

Prøvebanken (2021). *Folkeskolens afgangsprøve i matematik.* Danish Ministry of Education. Located 10.06.2021 at https://www.xn--prvebanken-1cb.dk/

Strømskag, H., & Chevallard, Y. (to appear). Elementary algebra as a modelling tool: A plea for a new curriculum [Manuscript submitted for publication].

Strømskag, H., & Chevallard, Y. (2022). *Conditions for revitalizing the elementary algebra curriculum*. [Paper presentation] 12th Congress of the European society for Research in Mathematics Education, 2-5 February 2022, Bozen-Bolzano, Italy.

Østergaard, K. (2021). Private conversation in relation to a presentation of the results of the national written examination.

Didactic transposition of fraction arithmetic in a Japanese overseas school: connecting a classroom episode to the curriculum

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The aim of this paper is to investigate how the directives for the teaching of fractions, set by the Japanese Ministry of Education, are transposed to the Japanese supplementary school in Denmark. We analyse an episode of teaching concerned with fractions in grade 5, based on a full-scale praxeological reference model of fraction arithmetic in Japan. This study is a part of my PhD project; therefore, it anticipates my first journal paper later.

Keywords: Japanese Arithmetic Curriculum, The Japanese Supplementary School, Teaching Fractions, The Anthropological Theory of Didactic, A Full-scale Praxeological Reference Model.

Introduction

Globalisation has caused expat communities to grow, and populous nations like Japan, France and the United States support "overseas schools" for children living abroad. The Japanese government, primarily the Ministry of Education, Culture, Sports, Science and Technology (MEXT) and the Ministry of Foreign Affairs of Japan, support pupils who have roots in Japan, living abroad, to be educated based on the Japanese national curriculum. They established Japanese schools and Japanese supplementary schools worldwide. There are 95 and 229, respectively, of these institutions, as of 15 April 2020 (MEXT, 2021). Supplementary schools are only open on Saturdays or during after-school hours, as a supplement for students who attend a regular national or international school. The teaching is limited to a few subjects, especially Japanese language and Mathematics.

We study here the mathematics teaching at a Japanese supplementary school in Copenhagen, Denmark, held Saturday in the morning. The school must cover the mathematical contents that pupils living in Japan learn in 136-175 hours per year, but has only 55.5 hours per year to do so. The school is renting rooms from a public Danish school. Unlike Japanese schools, the classrooms have smartboards instead of blackboards. Most teachers use the smartboard to open relevant pages in the Japanese textbook and write the answers directly on it. So, unlike what is common in Japanese mathematics teaching, the textbook is always displayed in front of pupils, and pupils do not take notes as in Japan. Teachers sometimes ask pupils to write down a definition of some mathematical term in their notebook, but due to time limitation and the students' difficulties with writing in Japanese, note-taking is downplayed at the school. Thus, the didactic infrastructure is very different from regular schools in Japan.

Certainly the pupils improve their Japanese language at the school, but it is also clear that they are not all at the level of children of their age in Japan. Furthermore, they attend other schools and have very limited time for learning mathematics following the Japanese national curriculum. To out knowledge, no research has examined the teaching and learning of mathematics under these or similar conditions.

Fraction arithmetic is one of the most problematic topics for pupils as well as for teachers (Winsløw, 2019). Moreover, pupil's knowledge of fractions in primary school is a strong predictor of their knowledge of algebra and overall mathematics achievement in high school (Siegler et al., 2012). In particular, knowledge of fractions at the age of 10 is known to be a strong predictor of mathematical achievement at the age of 16 (Winsløw, 2019). This motivated us to focus on the teaching of fractions in the Japanese supplementary school.

The aim of this study is to investigate how the directives for the teaching of fractions, set by the MEXT, are transposed to the Japanese supplementary school in Denmark.

Theoretical framework and research questions

The theory of didactic transposition in mathematics education begins with the assumption that mathematical knowledge lives in different institutions within a society, and is *transposed* (not merely transferred) between these institutions (Chevallard & Bosch, 2020). The *taught knowledge* in actual teaching at a school is the result of an internal transposition from the *knowledge to be taught* (the external transposition of *scholarly knowledge*). In other words, we need to identify the knowledge to be taught before analysing the actual teaching in a classroom. In another paper (Aoki & Winsløw, to appear), we have outlined the “global” praxeological reference model (PRM) which we have set up for the entire curriculum in primary school arithmetic in Japan.

We can now formulate the research questions of this paper as follows, given that we present here a case study based on one lesson on fractions at the Japanese supplementary school in Copenhagen:

RQ1. What mathematical praxeologies are developed by pupils during the lesson, in terms of the PRM of the corresponding theme in the national curriculum?

RQ2. How can the answers to RQ1 be related to the PRM of the entire sector of fraction arithmetic?

Data and Methods

The episode which we present here was related to the lesson which was taught on 30 October 2021 in a 5th grade with 3 pupils, and it was based on the part of chapter 10 in the textbook “5B Let’s Extend Addition and Subtraction of Fractions” (Fujii, T., & Majima., p.3-12). We choose a particular episode because it can show the advantage of using PRM of the entire sector of fraction arithmetic. The lesson lasted 90 minutes with a break in the middle. Pupils use the textbook published by TOKYO SHOSEKI, and their teacher more or less follow the teacher’s manual corresponding to the textbook. It should be noted here that in Japan, teachers’ manuals for textbooks are much longer than the textbook itself, and include many didactical ideas and suggestions based on previous experiments with the material, for instance in the context of lesson study.

The lesson was voicerecorded, transcribed into Japanese and then translated into English by the author. We also took field notes, as well as photographs of teacher's and pupils's written productions.

In order to answer RQ1 and RQ2, we conducted the analysis for actual lesson based on the full-scale praxeological reference model regarding fractions presented by Aoki et al. (to appear). One of the aims of this paper is to demonstrate how access to such a full-scale model helps analysing the didactical choices of both the teacher and the textbook, even when considering just a very short episode of teaching.

Findings

We now present the outline of the lesson and our analysis of an episode, based on the full-scale praxeological reference model.

Outline of the lesson

The teacher we observed has prepared presentation slides in advance to show on the smartboard, mainly consisting in pages from the textbook and some space where she can add hand written notes during the lesson. Pupils also have their textbooks open and write answers directly in the textbook. Only once do they take notes at the demand of the teacher. The teacher then interacts with the pupils to solve problems and exercises in the textbook.

Analysis of an episode

During the lesson, the teacher presents the students with the following task (t_1)*: Look for other fractions with the same magnitude as* $\frac{3}{4}$*, other than* $\frac{6}{8}$ *and* $\frac{9}{12}$. The teacher invites pupils to provide their answers orally. When we describe the teacher and pupil's utterances, we use letters such as T, P_1, P_2 and P_3. Here is an excerpt of the interaction between the teacher and pupils:

T:	Then, look for other fractions with the same magnitude as $\frac{3}{4}$, other than $\frac{6}{8}$ and $\frac{9}{12}$. For instance, you, P_1?
P_1:	$\frac{12}{16}$.
T:	$\frac{12}{16}$. What else?
P_2:	Can I? What I am thinking now..., $\frac{15}{20}$ maybe.
T:	How about you, P_3? Look for the same magnitude as $\frac{3}{4}$.
P_3:	Is it 16?
T:	$\frac{12}{16}$ has the same magnitude as $\frac{3}{4}$, right?
P_3:	But, how about 15?
T:	$\frac{15}{20}$. Can you make $\frac{?}{15}$? Is it a bit difficult?
P_2:	I guess it is correct? Multiply by 5.

T:	Multiply by 5?
P_2:	It multiplies by 5, right?
T:	In this case, multiply by 5. That is why $\frac{15}{20}$ is correct. I am now asking you to look for another fraction, P_3. Make (Find) a fraction with the same magnitude as $\frac{3}{4}$.
P_3:	But if so, should I add 20, +4 and 15 + 3?
T:	No, not addition.
P_2:	But, we can do it.
T:	Can we do it? Then, what happens if we add?
P_2:	Eh, $\frac{18}{24}$.
T:	If we add this, this will be $\frac{18}{24}$, won't it? But, is $\frac{18}{24}$ the same as $\frac{3}{4}$?
T:	Yes.
T:	Yes.
P_2:	So, we can do it.
T:	You are right.

According to above interaction between the teacher and pupils, we could consider that two techniques are developed by pupils. The first technique is the multiplicative technique (τ_1) from P_2's utterance "Multiply by 5" (implicitly, *both* numerator and denominator). The second technique is the Addition model (τ_2) from P_3's utterance. P_3 says "… should I add 20, +4 and 15 + 3?", and it can be interpreted as "in the sequence $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}$, the denominator and numerator increase by 4 and 3 respectively". Based on the PRM of theme in the curriculum, τ_1 is introduced for solving t_1, but τ_2 is not. Hence τ_2 is a new technique developed by pupils during the lesson, and (after some hesitance) accepted by the teacher as an alternative way to produce new fractions that are equivalent to a given fraction.

We may thus ask: why is the multiplicative model (τ_1) privileged by the textbook? In other words, why does it not introduce the addition technique (τ_2) for producing equivalent fractions? When we refer to our PRM of the entire sector of fractions, we could see that t_1 corresponds to a theme: Equivalence of fractions within the sector: fractions. When we look at this theme only, we could not see why τ_2 does not appear, as it is certainly a valid method to change fractional representation. But when we look further in our model, at theme on addition and subtraction of fractions, within the sector of operations with fractions, we find a plausible explanation. To add fractions with different denominators, we need to change denominators by a certain multiple, and the multiplicative technique is then much more efficient.

We note that in the episode, the teacher cannot explain to students why the technique τ_1 is to be preferred, and we do not know if the teacher is aware of the reason. In any case, as the teacher follows the textbook, the students will meet other tasks on equivalent fractions, like $\frac{2}{3} = \frac{?}{24}$, to reinforce the use of τ_1.

Conclusion

In this paper, we have investigated a case of how the directives for the teaching of fractions, set by the MEXT, are transposed to the Japanese supplementary school in Denmark. For this purpose, we have analysed an episode regarding fractions in grade 5 based on full-scale PRM. We have shown how the PRM of the entire sector of fraction allows us to analyse even small pieces of episodes from a broader perspective, especially in terms of connections across the external didactic transposition.

References

Aoki, M. & Winsløw, C. (to appear). How to map larger parts of the mathematics curriculum? The case of primary school arithmetic in Japan. Paper presented at CERME12, to appear in the proceedings.

Chevallard, Y., & Bosch, M. (2020). Didactic transposition in mathematics education. *Encyclopedia of mathematics education*, 214-218. Springer.

Fujii, T., & Majima, H. (2021). Atarashi sansu 512 sansuu no tobira [The new primary mathematics for grades 5B: The maths door]. Tokyo: Tokyo Shoseki Shuppan, 3-12.

Ministry of Education, Culture, Sports, Science and Technology-Japan. (2021). Japanese children studying abroad - Studying in the overseas educational institutions of Japan, global human resources & global teachers. Retrieved from https://www.mext.go.jp/a_menu/shotou/clarinet/002/001.htm.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy M. I & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science*, 23(7), 691-697.

Winsløw, C. (2019). What matters to students' success in mathematics? The case of fractions. *Journal of Teaching and Learning in Elementary Education (JTLEE)*, 2(2), 79-90.

ATD as an engine for evolution

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This article explains the first steps of a teacher's evolution from working with projects without following any theoretical frame to working with projects following the proposal of study and research paths from the Anthropological Theory of the Didactic as an instructional format within the paradigm of questioning the world.

Keywords: project-based teaching, study and research paths, statistics education, paradigm of questioning the world, secondary education.

1. Introduction

The first author is a Secondary School Mathematics teacher who has been teaching Statistics through projects in the last two years of Secondary Education. The initiative started six years ago as a need to engage highly unmotivated students with poor skills in mathematics. This experience has always been a success in terms of students' implications. Four years ago, apart from the motivation that the students found when carrying out a real and meaningful project, an external and also very inspiring incentive was found with the presentation of the projects to a contest. Some universities of Catalonia and Spain organise contests for Statistic projects in schools. Thanks to this participation, the first author could learn from the rest of the projects presented to the contest and kept improving the way projects were carried out. Last school year, the project "The noise and how it affects our health" won the first prize in the Catalan contest "*El planter de sondeigs i experiments*" as well as the Spanish Phase "*La incubadora de sondeos y experimentos*".

Although the projects had great success both among the students and in the contest, their design and implementation did not follow any theoretical frame. The teacher implemented them first with other colleagues but alone during the last year. She was supported by the school managers but not given any specific conditions. As a result, the teacher found project-based teaching very demanding and started having doubts about whether to go on with them or not. At this very moment, she started a PhD in mathematics education and was introduced to the anthropological theory of the didactic (ATD, Chevallard, 2015) and the proposal of study and research paths (SRPs). She then decided to make the instructional proposal evolve.

This paper presents the initial project-based instructional proposal from the ATD perspective, its analysis in terms of conditions implemented and constraints found, and the consequences in the design of an SRP to be carried out this year with the same group of students who won the last contest.

2. Analysis of the project from the ATD perspective

Freixanet et al. (2022) present a detailed description of the project "The noise and how it affects our health" from the perspective of the ATD. It relies on an a posteriori map of questions and answers – that we related to the SRP *chronogenesis* –, a description of the different activities and main agents – related to the *topogenesis* –, and the identification of the constraints that hindered the project implementation.

The results drawn can be summarised in the following main points:

- The initial questions had a strong generating character and corresponded to a real concern of today's citizens. Some of the students took them seriously. For instance, those who won the prize ended up preparing a website to inform about noise pollution and help measure the noise level of the visitors' living place. The web also includes videos to help the visitors visualize their data with an Excel sheet.
- The teacher took too much responsibility both in the design and the implementation of the project. It impeded students engage in some of the activities because they were already carried out by the teacher. It also ended up endangering the continuance of teaching through projects because of the unsustainability of the teacher's extra dedication.
- The media-milieu dialectic suffered from it. Most of the information was brought to the class by the teacher. Students only had to gather the empirical data (noise measures), prepare the questions for the survey, collect the answers, and summarise them under the teacher's guide. It was always the teacher who introduced the statistical techniques and notions that enabled the progress of the work.

- The questions-answer dialectic was also limited. The students did the activities by following the teacher's instructions and without having the whole perspective of the project. This can explain the different levels of engagement among them.
- About the individual-collective dialectic, students worked in teams of 3-4 and all the teams were addressing the same questions under the teacher's supervision. However, there were few opportunities for teams to collaborate, share their results and build on each other's results or proposals. The class run as a group of teams, not as a unique team divided into collaborative subteams. The contest imposed its conditions in this respect because it only accepts small team applications (between 1 and 5 students), not big group ones.
- The school institutional frame was not ideal either: the teacher worked individually with two classes of 30 students without the assurance of being able to access the computer room. The increase of dedication it demanded was too important to ensure its continuity. The school supported her – especially when the students won the contest – by did not take the proposal seriously enough to provide specific conditions for its implementation.

3. Design of the new project through SRP

The project for this academic year 2021-22 is being designed and implemented within the ATD and following the framework of the SRPs. In this article, we are presenting the design and the questions that have arisen during this process. At the conference, we will be able to present the results of the experience and the consequences drawn from them.

3.1. Institutional frame

The project is being carried out again by one teacher and two classes of 30 grade 10 students (15-16 years old). In this grade, the mathematics subject typically has four sessions of 50 minutes per week. However, each school has the possibility to add some hours. In our case, it was decided to devote one extra session per week to the statistics project work. During these sessions, the availability of the computer room is assured. The students work in teams of 4/5 and are the same students from the previous project, so they are already aware of the procedure of working with projects. During the first part of the course (September-December), the teacher proposed specific statistical activities focused on some important questions and techniques like "Is your dice tricked?" to study univariant descriptive statistics and "What do you want to know about your classmates?" to introduce how to design a survey properly to guarantee a successful analysis and different types of data representation. The SRP will be implemented during the second part of the course (January-June).

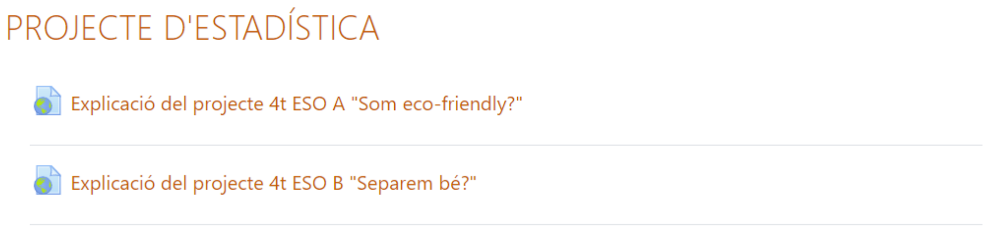
3.2. Preparation of the didactic infrastructures for the collaborative work

The teacher prepares the main documentation (objectives, learning outcomes, initial timing, a priori question-answer map from the brainstorming, and assessment) and the Moodle course as a workbench (see Figures 1 to 8). It is the same for both classes and, at the moment of writing this article, it has the following structure:

- General documentation about both projects.
- Tasks planning: Phase 1. Coinciding with the first three sessions it is organised by the teacher, whereas from Phase 2 until the end of the project it is decided by the students.

- Partial submissions: the teacher decides to divide the final product into partial submissions so that the students keep the work done while doing the research. The teacher thinks it is also a way for the students to stay connected with the project and not lose the main purpose of it.
- Draft of questions-answers maps.
- Sample of the *padlet* they can use to gather the information
- Information sharing: it contains a link to a Google Document where the students will share the main aspects of the information found and a document that the teacher creates as a result of the information sharing process carried out in class.
- Peer-to-peer and self-assessment: a section with a link to a Google Form where the students evaluate themselves and their teammates.
- A section for each team in which they have a link to the project diary, a document in which the students have chosen a task they are responsible for within their team, and to the *padlet* of their own team.

All the sections are common, and the students share all the information apart from the last one. This structure is not final, it keeps changing and increasing as the project advances.

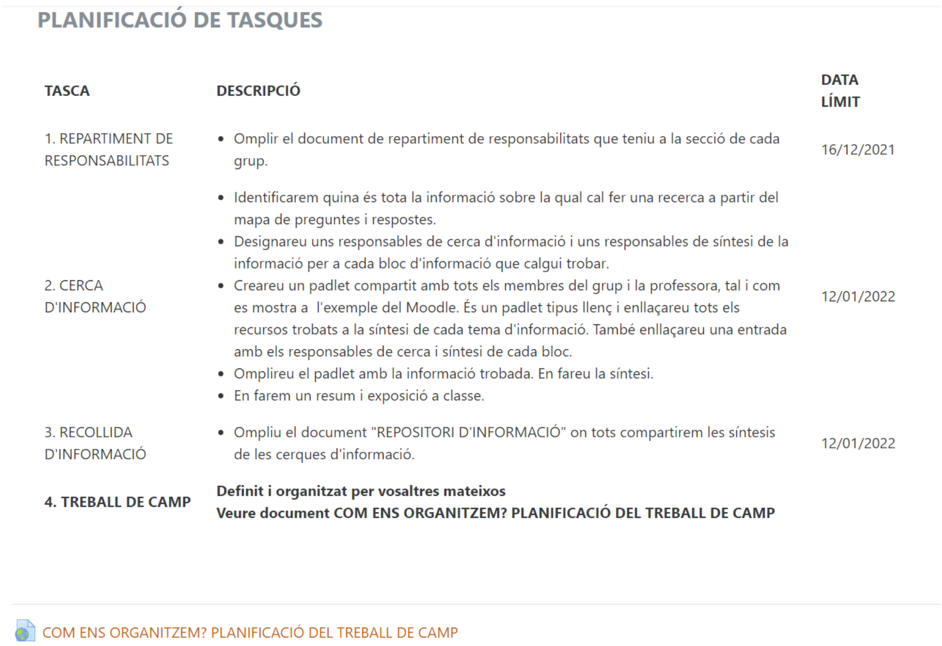


PROJECTE D'ESTADÍSTICA

Explicació del projecte 4t ESO A "Som eco-friendly?"

Explicació del projecte 4t ESO B "Separem bé?"

Figure 1: General documentation about both projects



PLANIFICACIÓ DE TASQUES

TASCA	DESCRIPCIÓ	DATA LÍMIT
1. REPARTIMENT DE RESPONSABILITATS	• Omplir el document de repartiment de responsabilitats que teniu a la secció de cada grup.	16/12/2021
2. CERCA D'INFORMACIÓ	• Identificarem quina és tota la informació sobre la qual cal fer una recerca a partir del mapa de preguntes i respostes. • Designareu uns responsables de cerca d'informació i uns responsables de síntesi de la informació per a cada bloc d'informació que calgui trobar. • Creareu un padlet compartit amb tots els membres del grup i la professora, tal i com es mostra a l'exemple del Moodle. És un padlet tipus llenç i enllaçareu tots els recursos trobats a la síntesi de cada tema d'informació. També enllaçareu una entrada amb els responsables de cerca i síntesi de cada bloc. • Omplireu el padlet amb la informació trobada. En fareu la síntesi. • En farem un resum i exposició a classe.	12/01/2022
3. RECOLLIDA D'INFORMACIÓ	• Ompliu el document "REPOSITORI D'INFORMACIÓ" on tots compartirem les síntesis de les cerques d'informació.	12/01/2022
4. TREBALL DE CAMP	**Definit i organitzat per vosaltres mateixos** **Veure document COM ENS ORGANITZEM? PLANIFICACIÓ DEL TREBALL DE CAMP**	

COM ENS ORGANITZEM? PLANIFICACIÓ DEL TREBALL DE CAMP

Figure 2: Tasks planning

ENTREGUES

26-27 de gener de 2022

1. Informe parcial:
    1. Estructura del document
    2. Índex
    3. Introducció
    4. Objectius del projecte
    5. Síntesi de la informació documental trobada
    6. Temporització del projecte: quan farem les mesures, quan les campanyes...
    7. Webgrafia
2. Mapa de preguntes i respostes: cada grup el seu i actualitzat
3. Diari del projecte al dia
4. Padlet definitiu

2 de març

1. **Informe parcial:**
    1. **Estructura del document**
    2. **Índex**
    3. **Introducció**
    4. **Objectius del projecte**
    5. **Síntesi de la informació documental trobada**
    6. **Temporització del projecte: quan farem les mesures, quan les campanyes...**
    7. **Mesures preses i inici de l'estudi estadístic**
    8. **Webgrafia**
2. **Mapa de preguntes i respostes: cada grup el seu i actualitzat**
3. **Diari del projecte al dia**

Entrega 1

Opened: dimecres, 19 gener 2022, 00:00
Due: dimecres, 26 gener 2022, 23:59

Entrega 2

Opened: dimecres, 19 gener 2022, 00:00
Due: dimecres, 2 març 2022, 23:59

Data límit:

4t ESO A: 1 de març 23:59

4t ESO B: 2 de març 23:59

Figure 3: Partial submissions

MAPA DE PREGUNTES I RESPOSTES

Figure 4: Questions and answers map sample

DISPOSEM DE FONTS DOCUMENTALS OFICIALS? DE QUINA INFORMACIÓ DISPOSEM?

Figure 5: Padlet sample

RECULL D'INFORMACIÓ

Aquest enllaç conté un document que és un quadre-resum de tota la informació trobada. El farem servir per la posada en comú a l'aula els dies 12 i 13 de gener de 2022. Cal omplir-lo amb el resum que heu fet de cada pregunta.

Vigileu de no repetir preguntes!

https://docs.google.com/document/d/1-NBAda6n9QwrnTYiH0aAZ3m4ckfVXyARU1lTxfkUCGU/edit?usp=sharin

Resum del recull d'informació de tots els grups

Figure 6: Information gathering

AVALUACIÓ

Avaluació i coavaluació 4t ESO A

Avaluació i coavaluació 4t ESO B

Figure 7: Self- and co-evaluation

Diari del projecte Restringit No disponible llevat que es donin totes les condicions següents: Pertanyeu al grup **B1**

Repartiment de responsabiltats Restringit No disponible llevat que es donin totes les condicions següents: Pertanyeu al grup **B1**

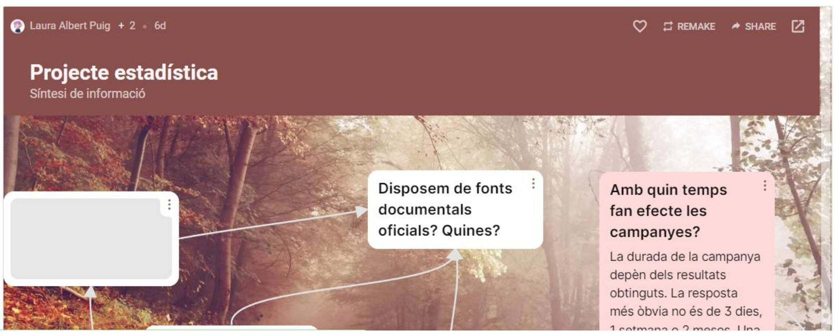


Figure 8: Team section: project diary, responsibilitie distribution and team padlet

3.3. Assessment

The assessment of the project will be divided in three aspects:

- Final report of the project (40%)
- Video explaining the project and presenting the results and conclusions (40%)
- Attitude (20%): this item will include the peer-to-peer assessment (40%), the self-assessment (10%) and the teacher's assessment (50%)

In the first two items, the following aspects will be assessed: objectives of the study, description of the process, quality of the statistical analysis, results, and conclusions.

In the third item (attitude), students assess their team members and themselves through a Google Form. This assessment, at this point of the project, has been defined and prepared by the teacher. It follows a rubric with the following items: collaboration within the team, conflict resolutions and individual responsibility. Each item is graded with four options: good, fair, weak and no evidence.

4. Implementation of the project: start-up

The implementation of the project contains 3 phases.

Phase 1: Choice of the generating question and first explorations

Phase 2: Fieldwork: data gathering and campaigns

Phase 3: Data analysis and conclusions

4.1. Phase 1: Choice of the generating questions and first explorations

The teacher organises a brainstorming activity with the students in class, tells them that a new project will be carried out this year and proposes three topics to discuss: Do we recycle properly?, Are we too connected? or something related to "water".

This brainstorming activity was carried out in both classes, and it led us to choose the topic the students preferred: "Do we recycle properly?". Although new questions appeared, we decided we could do more than one study using the same gathered data. So, one class would answer the question "Are we eco-friendly?" and the other, "Do we sort waste properly?" (see Figure 9).

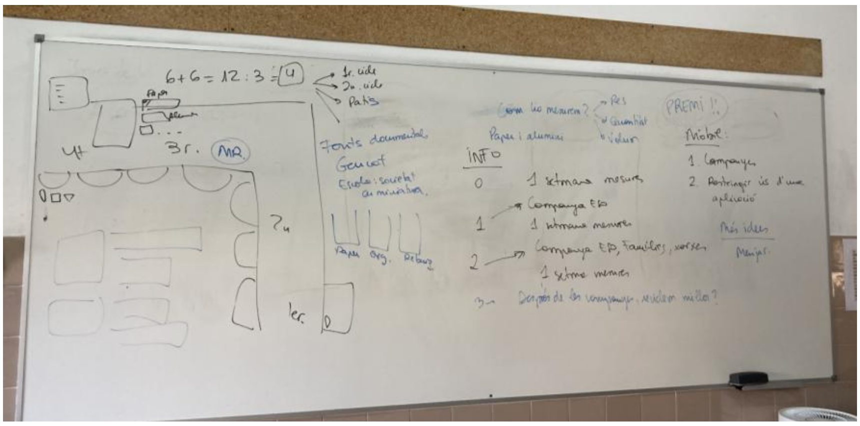


Figure 9: Whiteboard of one class after the brainstorming process

The teacher decided to be responsible only for the organisation of the project and not for the research of the information, which is left entirely to the students’ responsibility. The teacher also decides to group the students into teams of four and include one of the five students who won the statistical contest last year as spreading agents in some of the teams. This means that 20 students will have direct access to the experience of the extra work those students did.

In the first explorations, the teams do the research and gather it in their team *padlet*. This *padlet* has the “Canva style” (see Figure 10). The students post the question they want to find information about, and they link all the documents and websites they find regarding this question. After all the information is uploaded, they also create a post with a summary of it all and another one explaining how they have organised themselves as a team (who has searched for the information and who has summarised it). All the posts are linked to the corresponding question.

After the synthesis of the information is done, all the teams share the results in the Google Drive document of the Moodle course with the rest of the teams. Figure 4 shows how the information is shared: the question is written and each team (A1, A2, ..., A7) write down the findings. The students share the results in class and comment on them with the teachers. The pictures in Figure 5 show examples of the information shared and agreed upon in class. The teacher then makes a document synthesising the information and proposals from both classes (Figure 13).

As we can see, the *pedagogical and didactic infrastructures* elaborated and established by the teacher is not trivial. They are supported by a variety of online tools and applications that need specific skills to be organised and managed by the teacher and the students. Some of them are inherited from the project work done the previous year (the *padlet* and the Google Drive shared documents) but others are new.

Figure 10: Padlet containing the information gathered by one team

| RECULL D'INFORMACIÓ DE TOTS ELS GRUPS

En aquest document, recollim la informació trobada per tots els grups en la fase de recull d'informació.

SOM ECO-FRIENDLY?

PREGUNTA	A1	A2	A3	A4	A5	A6	A7
Com ho farem arribar als nostres companys?	- Posar cartells al voltant de Rubí. - Podem anar per les classes anunciant el que fem i conscienciar una mica a la gent. - Podem repartir publicitat amb fullets a la sortida del col·legi. **- Això és una mica més complicat, però ens sembla una bona idea:** Fer o crear algun joc per així	-Per correu -Cartells a les parets dels passadissos -Repartir circulars -Posar-ho a clase -Fer una presentació als companys.	-Fent xerrades als cursos, amb presentacions -Mitjançant el correu electrònic enviar a pares i alumnes circulars amb informació sobre les campanyes que es faran. -Posant cartells informatius, com infografies o posters a les classes i als passadissos -Fer tallers a	-Pósters físics o virtuals. -Correu electrònic. -Presentacions.	- Enviant correus als alumnes i pares. - Penjant pòsters i cartells per tota l'escola. - Fent xerrades a les classes. - A través de les xarxes.	-Correu electrònic	-Mitjançant una campanya d'anuncis i cartells. -Enviant correus electrònics als pares dels alumnes de l'escola.

9

Quins materials tindrem en compte?	Paper d'alumini, cartró, paper, plàstic i resta organica.	-Plàstic -Cartró -Paper d'alumini -Resta -Paper	-Paper d'alumini -Plàstics -Resta organica -Paper	-Cartró. -Alumini. -Plàstic. -Paper. -Resta orgànica..	-Paper d'alumini. -Paper de cuina. -Plàstics. -Resta. -Rebuig orgànic.	Material reciclatge	Plàstic Alumini Cartró
Quines dades volem recollir?	-Nivell de reciclatge -Residus generats a l'escola	-Nivell de reciclatge a l' escola -Quantitat de residus que generem a l' escola	-Quantitat de residus generats a l'escola -Si fem correctament la recollida selectiva -Quin material es el que mes es llença a l'escola -conciencia dels alumnes sobre un	-La quantitat de kg que hi ha en cada tipus d'escombraries (cartró, plàstic i orgànica) dins de les classes de ESO, l'aula groga i informàtica. -Farem una enquesta preguntant si consideren que	- Si els alumnes llencen la brossa al contenidor corresponent. -La quantitat de brossa que llencem al pati i a les classes de l'ESO.	-Per a alguna de les nostres ciutats costaneres, moltes de les nostres deixalles acaben a l'oceà i mars, i això inclou una gran quantitat de materials	-Si fan una bona recollida selectiva. -Pes de l'escombreria de cada paperera a les classes.

Figure 11: Document where the synthesised information is shared by all the teams

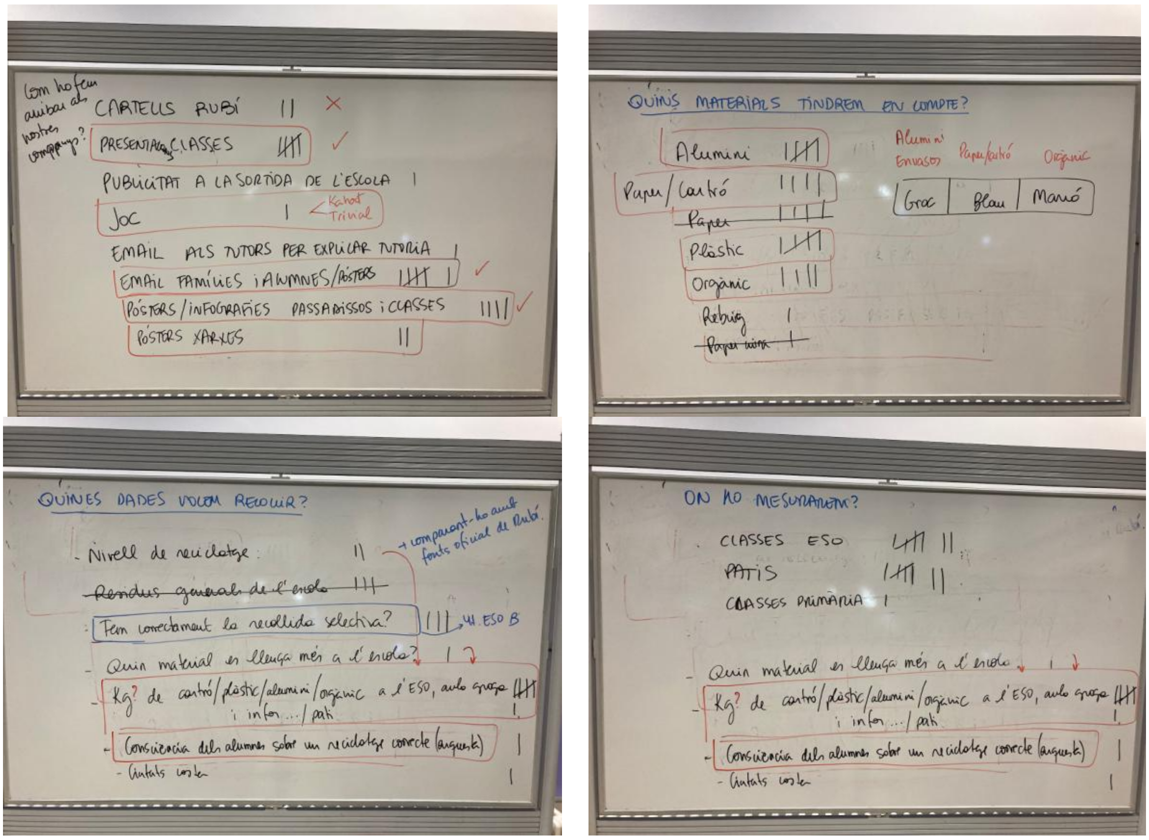


Figure 12. Images of the whiteboard in the process of sharing the information found.

RECULL D'INFORMACIÓ DE LA RECERCA

A continuació, es presenten els resultats de la posada en comú de la recollida d'informació de la recerca de la mateixa a les classes de 4t ESO A i B:

Com ho farem arribar als nostres companys?

4t ESO A	4t ESO B
• Presentacions a les classes • Joc (Kahoot, Trivial) • Correu a les famílies i als alumnes • Pósters/infografies als passadissos de l'escola i/o a les classes • Pósters a les xarxes	• Xerrades dinàmiques a les classes • Joc • Posters a l'escola (pregunta: si fem cartells, no estem contaminant més? Resposta: si el resultat és positiu, val la pena) • Xarxes socials (visual i/o interactiu) • Mural a l'entrada • Cartells als contenidors de l'escola

Quins materials tindrem en compte? (4t ESO A)

- Alumini i envasos (cal definir si es fa junt o per separat)
- Paper/Cartró
- Orgànic

Figure 13: Summary of the information found by all the students after sharing it in class

4.2. Phases 2 and 3: Fieldwork and analysis

Phases 2 and 3 are still in process but the organisation is explained as follows. To start with Phase 2, the teams make proposals on how to carry out the fieldwork. This work has been divided into two main activities called: "Data gathering" and "Campaign". In this phase, the initial teams are dissolved, and each student decides what group they want to join according to the activities chosen. Some students volunteer to be the leaders of these groups. The leaders are responsible for:

- Tasks distribution and follow-up among the students in their group.
- Communication with the leaders from the same group in the other class.
- Communication with the leaders from the other groups.
- Communication with the teacher.

This phase is intended to be led by the students with the guidance of the teacher.

To carry out Phase 3, the students will return to their initial teams and will analyse the results of the fieldwork to prepare the final product of the project.

5. Conclusion

This paper presents a work in progress. It will be finished at the time of the conference and more results will be provided. We can here indicate the research questions we want to explore with the experimentation.

The first one relates to the description of the SRP using the Herbartian schema and its dialectics. How can we provide a detailed account of the inquiry process? What elements must be pointed out? How can we report about the dialectics? In what terms? The methodology we have used in other research works (Barquero & Bosch, 2015; García et al., 2019) considers a description of the questions-answers dialectic that runs through the SRP (*chronogenesis*), completed with the dynamic of the media-milieu dialectic (*mesogenesis*) to explain what originates the questions and how new information and new tools are incorporated to the milieu. Finally, the sharing of responsibilities between the teacher, the students' teams and the group of students can be addressed in terms of the individual-collective dialectic (*topogenesis*). In the case here considered, there are two classes carrying out the SRP in parallel and exchanging information and data, which introduces some complexity but also generates a more interesting dynamic.

These questions must be related to the one of the pedagogical and didactic infrastructures provided by the teacher, their effectiveness as conditions to run the SRP and the constraints they necessarily posed: any condition enables or facilitates some activities while hindering or impeding others. The interaction between pedagogical and didactic infrastructures can be especially promising, especially when linked to the kind of works studied by the students, the new tools and knowledge developed during the inquiry, the final answers produced and their fate in the school life or beyond.

The account presented here mainly corresponds to the teacher's perspective, her decisions and description of the class activities. Gathering evidence about the students' productions through the activities carried out in Moodle, the reports submitted, some interviews and a questionnaire proposed at the end of the SRP is necessary to contrast the two positions of the inquirers.

A final crucial point is the evolution of the teacher's didactic praxeologies – both its praxis and especially its logos – due to her immersion in the ATD. It can shed light on the institutional conditions needed for the dissemination of SRPs and more generally instructional proposals close to the paradigm of questioning the world.

6. References

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: from fundamental situations to study and research paths. In A. Watson & M. Ohtani (Eds.), *Task Design in Mathematics Education. An ICMI Study 22* (pp. 249-272). Springer.

Chevallard, Y. (2015). Teaching Mathematics in tomorrow's society: a case for an oncoming counter paradigm. In S. Cho (Eds.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer International Publishing.

Freixanet, M. J., Alsina, M., Bosch, M. (2022). How does noise affect our health? Analysing a project-based activity in Statistics at the secondary level. *Proceedings of CERME12* (in press).

García, F. J., Barquero, B., Florensa, I., & Bosch, M. (2019). Diseño de tareas en el marco de la Teoría Antropológica de lo Didáctico. *AIEM - Avances de Investigación en Educación Matemática, 15,* 75–94.

Didactic transposition of statistics at university level: a study design

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Abstract. *We present an outline of a research plan for analysing the didactic transposition of applied statistics at the university level. This work is part of a doctoral thesis of the first author and is intended to provide an insight on the didactic transposition in the process from the scholarly knowledge of statistics to the knowledge taught in class. We propose qualitative research by conducting semi-structured interviews with university teachers, researcher and practitioners in statistics from different countries. By analysing different practices of statistics teaching, we aim to detect the crucial steps in the process of the didactic transposition and therefore benefit from it when designing, implementing, analysing and developing instructional proposals as study and research paths (SRPs) into university statistics teaching.*

Resumen. *Presentamos un esquema de un plan de investigación para analizar la transposición didáctica de la estadística aplicada en el ámbito universitario. Este trabajo forma parte de un trabajo de doctorado de la primera autora y pretende aportar una visión sobre la transposición didáctica en el proceso desde el saber sabio de la estadística hasta el saber enseñado. Se propone una investigación cualitativa mediante la realización de entrevistas semiestructuradas a profesores universitarios, investigadores y profesionales de la estadística de diferentes países. Mediante el análisis de las diferentes prácticas de la enseñanza de la estadística, pretendemos detectar los pasos cruciales en el proceso de la transposición didáctica y, por tanto, beneficiarnos de ella a la hora de diseñar, implementar, analizar y desarrollar propuestas como recorridos de estudio e investigación (REI) en la enseñanza universitaria de la estadística.*

Introduction

Statistics as a professional area started experiencing a tremendous change with the technological development during the 1990s. The notion of big data brought out both the simplification as well as the complication for statistical work. New technologies, among them the internet, allowed fast, direct and massive data collection, while the spread usage of the programming languages acted as tools for efficient and accurate data analysis. However, the bigger the possibilities were, the demand for understanding certain statistical results grew accordingly. Numerous researchers and data scientists coined the term "data literacy" linked to "the ability to identify, retrieve, evaluate and use information to both ask and answer meaningful questions." (McAuley et al., 2010, p. 89) or the ability of the non-specialists to learn about data science and apply the gained knowledge to solve problems (Pedersen et al., 2019).

The development of the statistics profession brought with it a questioning of the demands in knowledge and skills – the praxeological equipment – and how to satisfy them in university courses.

Nolan and Temple Lang (2010) pointed out the necessity to expand the computational ability for supporting statistical inquiry and proposed working on real computational problems that arise from data acquisition, statistical analysis, and reporting. Nevertheless, implementing changes into statistics curricula did not seem appropriate if it meant simply adding extra activities without revising the current ones: "Instead of trying to fit bits and pieces into a crowded curriculum, statisticians must take the opportunity to be bold and design curricula from scratch that embrace new and innovative topics and paradigms for teaching." (op. cit., p. 28)

Designing the curricula "from scratch" may be a way of claiming for a development of the transposition work in statistics (Chevallard, 1985; Chevallard & Bosch, 2020). We know that, due to the strong inertia of university mathematical programmes, external transposition processes seem to take a long time to run – or at least to get noticed in the classroom (Bosch et al., 2021). However, the case of statistics does not seem to follow the same evolution. It seems that the social need for competent data scientists and the easy dissemination of online instructional resources represent too great a pressure to maintain the traditional knowledge organisation. To this must be added the facilities to incorporate technologies and statistical software in the classroom.

Our goal is to go deeper into this didactic transposition process that is currently taking place before our eyes and in which we are ourselves involved as lecturers. We are presenting here the part of the study that wishes to approach the awareness of statisticians and university professors from different countries about the evolution of the field and its consequences in their statistics teaching practices.

Research questions and methodology

Our main research questions are:

> RQ1: How do statistics teachers and professionals conceive the change of statistics as a profession?
>
> RQ2: How does the change in the profession influence university teaching of statistics?

We aim at collecting and analysing data from semi-structured interviews with statisticians teaching in university degrees for non-mathematics and non-statistics students. We want to get to know about different experiences and interpretations of how applied statistics is being taught nowadays and what changes have been observed during the last decade.

According to the aims and to answer the mentioned research questions, the interview was designed starting with an introductory description of the study objective, asking for the interviewee's academic formation and an overview of their work experience. The interview script continues into five sections:

1. Stating the *statistics and statistics teaching essentials* according to different interviewee's roles: as a teacher, as a researcher and as a professional in statistics.
2. *Chronogenesis* of the statistics courses the interviewees are teaching or have taught in the recent past at the university level, asking about: content structure and timing, software, bibliography; and the relation between the course they teach to the other courses of statistics in the same degree (e.g., the connection of introductory and advanced statistics within the same degree).

3. *Mesogenesis and topogenesis*: the didactic forms of their teaching, such as the traditional lectures, case studies, projects, article analyses, etc. Description of the interviewee's teaching method and the resources they have for implementing different didactic devices to the classes.
4. *Didactic technology and ecology*: justification of the teaching method and the autonomy to change it; detecting the constraints coming from different levels of the scale of didactic codetermination.
5. *The final reflection* on the interviewee's experience of teaching statistics, having discussed the abovementioned topics during the interview, and a comparison between their professional practice in statistics and the teaching of statistics.

Hypotheses for the study

In the following sections, we describe in detail the interview script (questions denoted as Q) and the hypotheses (denoted as H) in terms of the reasonings that we expect to obtain.

Statistics and statistics teaching essentials

At the beginning of the interview, we state the academic formation of the interviewee (whether it is in pure or applied mathematics, engineering, business or anything else) as well as their work experience and interest in scientific research. This might later show common inclinations towards certain ideas on how statistics should be taught. However, the introduction of the subject is also to bring awareness of the different roles a statistics teacher can have in relation to the statistics as a profession and the teaching of it.

Q1: "What is essential in statistics for you and needs to be taught? Consider your roles as a teacher, a researcher and a professional in stats."

H1: Generally speaking, university teachers and professionals in statistics are confident when talking about statistics since they master the statistics praxeologies and are successful in defending the core of the field. However, it is uncommon to critically think about the teaching of the subject. We are curious about the perspective that our interviewees take on statistics from different points of reference and what language they use to elaborate on it. We also wish to know their predisposition for an update of the transposition process. Therefore, to finish off with the introduction, we ask about the change of the essentials:

Q1.2: "What is essential now and was not before (in statistics or the teaching of statistics)?"

This question aims for the more experienced teachers to contrast the evolution of their own teaching, while in case of younger teachers, a comparison of what they were taught as students and what they teach.

Chronogenesis

The next part considers the general content organisation for the introductory statistics course:

Q2: "What content do you follow in your teaching of statistics? Do you use any software in class? Why do you organise your teaching the way you do? What bibliography do you rely on and recommend to your students?"

In this section, we discuss about the in-class praxis of the teacher to confirm the common statistics course content organisation or to get insights into the reasons standing behind possible unexpected information we get. In any case, we expect to mainly get answers concurrent to the following:

H2.1 – content: We expect some general terms to be mentioned, such as: descriptive statistics, both graphical and numerical and their comparison in terms of importance; probabilities, whether introduced by the simple distributions such as the normal and the binomial one or not at all; problems of sampling and inference, do they matter and how are they used as a connection between probabilities and hypothesis testing; and finally, the hypothesis testing and the relevance of introducing them in applied statistics course. However, we also expect some interviewees to present a newer version of the content, based on software use and the searching, cleansing and handling of data.

H2.2 – raison d'être of the list of contents: We expect interviewees to refer to the tradition or the logical organisation of the concepts to justify the classic organisation and to the software structure and data management professional activities for the updated one.

H2.3 – bibliography: We wonder if they consider the available bibliography to be sufficient and of good quality for the course they are teaching, and if they are supporting it with exercises prepared by themselves.

To sum up the chronogenesis section, we wonder about the comparison between the current course setup and its organisation 10-20 years before.

Q2.1: "What has changed in regard to the software usage?"

Have there been any changes, and if so, what is the purpose of the software.

H2.1.1: Is the statistics course the same as before, just with a software added as an additional activity, or the whole concept of introductory statistics has changed due to the software implementation?

Finally, we are curious about the relation between the introductory and advanced statistics courses within the same degree programs for it to possibly bring out some hidden issues or options that are influencing the organisation of the courses.

Mesogenesis and topogenesis

Mesogenesis addresses the milieu exploited when teaching statistics, and topogenesis the responsibilities assigned to the students (didactic contract).

Q3: "How do you organise your classes in terms of lecture form? Why do you do it that way and what resources do you have/use?"

H3.1 – lecture form: We expect a description of the course the interviewees teach, whether it is arranged in a traditional lecture-tutorials mode, or implementing case studies, projects or similar.

H3.2 – resources: Rationalising software implementation and the usage of it throughout the course might bring out some interesting findings, whether it regards its exploitation for most of the course activities or just as an accessory additional to the traditional paper and pencil exercises.

What we would like to focus on is the purpose of the software that is used in today's statistics classes. More specifically, what software are used (if they are used) and in what kind of activities (only for book example exercises or for cleaning the raw data as well).

The context of the topogenesis, the sharing of the responsibilities in the established didactic contract between the teachers and the students, could be analysed twofold. The connections to the topogenesis could be drawn according to the lecture forms and the usage of the resources, and additionally referring to the initial part of the interview related to the different perspectives that the interviewees take towards statistics (as a teacher, as a researcher and as a professional in statistics).

Didactic technology and ecology

About the justification of the teaching method, we are continuing with the questioning the influences, internal and external to the teacher, that shape the teaching as it is.

> Q4: "Why do you teach the way you do? Have you considered changing your teaching method? Do you feel any constraints coming from the institution, the students' autonomy, time or curriculum organisation, lack of technological resources, etc.?"

H4: We aim to both detect the constraints that university teachers of statistics observe in their work, as well as the criticism and ability to acknowledge and express such observations.

This section is to be a culmination point before the final reflection on the issues discussed, completing the conversation on statistics teaching, its evolution and the connection to the sphere of the statistics practitioners. Additionally to the conducted interviews, a study of the interviewees' available course syllabi might enrich the analysis of the logos part of the didactic organisation of the statistics teaching.

Final reflection on the interview and the teaching

The final reflection is a chance for the interviewees to contemplate their answers to the questions and maybe phrase some parts differently, having on mind some of the perspectives they might have not considered prior to the interview.

> Q5: "What Statistics that you teach is coherent with what you think Statistics is?"

By asking Q5, we intend to get the last revision on H5: whether the teachers consider their teaching to be separated or united to their research/professional practice and if they have any thoughts on what is essential for the profession and should be in the university statistics curriculum.

Further implications of the study

The study on didactic transposition of university statistics emerged from the doctoral research of the first author about the ecology of study and research paths (SRPs) in university statistics. Some of the identified conditions and constraints come from both the lower levels of the scale of didactic codetermination (discipline, domain, etc.) and the highest ones (such as school and society). They appear to be linked to the current evolution of the scholarly knowledge and a more and more urgent need to update the teaching of statistics. Better knowing the situation of the noosphere of the university educational system seems critical to overpass the thin line between an actual praxeological change and a mere cosmetic arrangement.

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References

Bosch, M., Hausberger, T., Hochmuth, R., Kondratieva, M., & Winsløw, C. (2021). External didactic transposition in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *7*(1), 140-162.

Chevallard, Y. (1985) La Transposition Didactique. Du savoir savant au savoir enseigné, 2nd edn 1991, La Pensée sauvage, Grenoble [Spanish translation: Chevallard Y (1997) La transposición didáctica. Del saber sabio al saber enseñado. AIQUE, Buenos Aires]

Chevallard Y., & Bosch M. (2020). Didactic Transposition in Mathematics Education. In: Lerman S. (Eds.) *Encyclopedia of Mathematics Education*. Springer, Cham.

McAuley, D., Rahemtulla, H., Goulding, J., & Souch, C. (2010). How open data, data literacy and linked data will revolutionise higher education. In L. Coiffait & J. Hill (Eds.), *Blue Skies: New thinking about the Future of Higher Education. A collection of short articles by leading commentators* (pp. 88-93).

Nolan, D., & Temple Lang, D. (2010). Computing in the Statistics Curricula. *The American Statistician, 64*(2), 97–107.

Pedersen, A. Y., & Caviglia, F. (2019). Data literacy as a Compound Competence. In T. Antipova & A. Rocha (Eds.), *Digitals Science. DSIC18 2018. Advances in Intelligent Systems and Computing* (Vol. 850, pp. 166-173).

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