The 7th conference on the ATD (CITAD7)
June 19–23, 2022; CRM, Barcelona, Spain.

The Paradidactic

A Theory of Noospheres

Koji Otaki, Hokkaido University of Education, Japan.

Contents

Introduction: dualizing the didactic

1/7

Didactic continent

Didacticians live in the *didactic continent*.

It is a "spatial" metaphor for the world where didacticians are involved.

This continent consists of many areas of *didactic reality*.

* The didactic reality of mathematics
* The didactic reality of Catalan
* The didactic reality of football, etc.

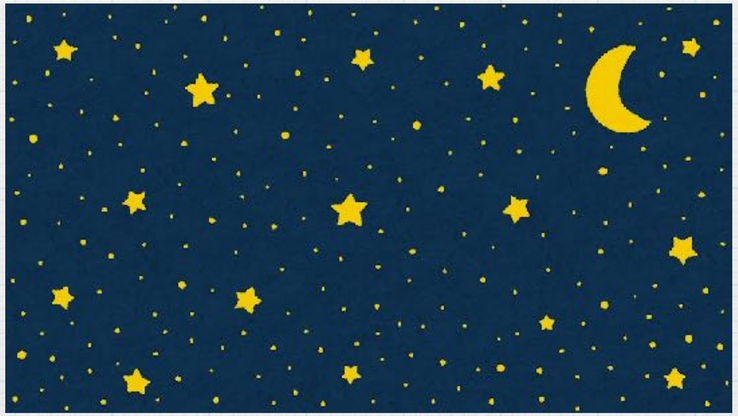
Two periods of “didactic” time

Let me go further with the metaphor of “continent”.

There are two periods of time .

* The period of *school*: related taught knowledge

* The period of *noosphere*: related to knowledge to be taught



The discovery of noospheres

Noosphere: people who think about teaching
e.g., curriculum-developer and parent

The notion of noosphere has brought about a breakthrough to didactics.

Problematizing the field of knowledge to be taught.

Theorizing the noosphere

"The time of ideas often passes much slower than the time of people." (Bosch & Gascón, 2006, p. 52)

On the one hand, the *didactic transposition theory* has developed to the ATD.

On the other hand, the notion of noosphere is keeping its original "form" without any explicit elaboration on it.

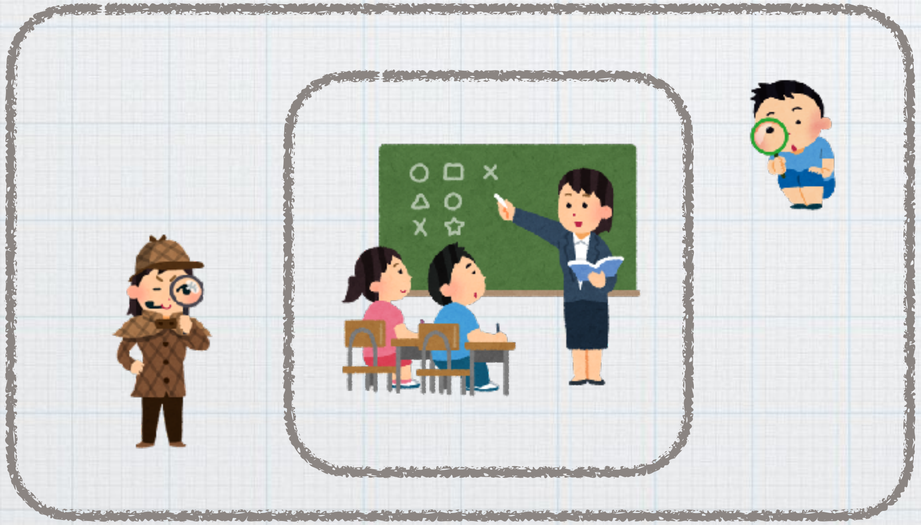
Aim of this talk

To theorize didactic reality involved in noospheres as much as possible.

A keyword for that is the adjective *paradidactic*, which is a much younger notion than the noosphere.

My summary of the great achievement by the didactic transposition theory: it has considerably expanded the research field of didactics by *dualizing* the “didactic” into the (narrowly) didactic at school and the paradidactic at noosphere.

From noospheres to paradidactic systems



2/7

Paradidactic system

Any scientific field has its own "system" to study, model, and understand.

Didactic system $S(X, Y, \heartsuit)$: A kind of social systems, in which someone X with the help of someone Y studies something ♥.

Paradidactic system $S(\dot{X}, \dot{Y}, S(X, Y, \heartsuit))$: We can consider about the possibility that ♥ is also a didactic system for schoolteachers $\dot{X}$.

Examples

* **Collective plan and reflection in a small lesson study:** $S(\{\dot{x}_1, \dot{x}_2, \dot{x}_3\}, \dot{y}, S(X, \dot{x}_1, \heartsuit))$.
* **Individual plan of and reflection on a lesson:** $S(\dot{x}, \emptyset, S(X, \dot{x}, \heartsuit))$.
* **Collective plan and reflection in lesson simulation:** $S(\{\dot{x}_1, \dot{x}_2, \ldots\}, \dot{y}, S(\{\dot{x}_1, \dot{x}_2, \ldots\}, \dot{x}_1, \heartsuit))$.
* **Observation of a lesson:** $S(\dot{X}, \emptyset, S(X, y, \heartsuit))$.
* **Teachers's "transcendental" observation during a lesson:** $S(y, \emptyset, S(X, y, \heartsuit))$.

Anthropological caution

Everything could be “didactic” in some sense.

Paradidactic systems are didactic systems of a special kind.

In this sense, paradidactic reality is only a part of the vast field for the application of the ATD.

But, on the other hand, its “nested-ness” requires specific theorization.

The formalization of noospheres

Let me formalize the notion of a noosphere with some letters: $\dot{N}$ (a noosphere), x (a person), Σ (a school), and $\check{S}$ (a society).

$\dot{N} := \{x \in \check{S}_\Sigma \mid R(x, \Sigma) \neq \emptyset\}$. ($\check{S}_\Sigma$ means "$\check{S}$ involving Σ")

The notion of "a noosphere" indicates *a noosphere of a given school in a certain society*, which can be denoted by $\dot{N}_\Sigma$.

“Noospheric” professions

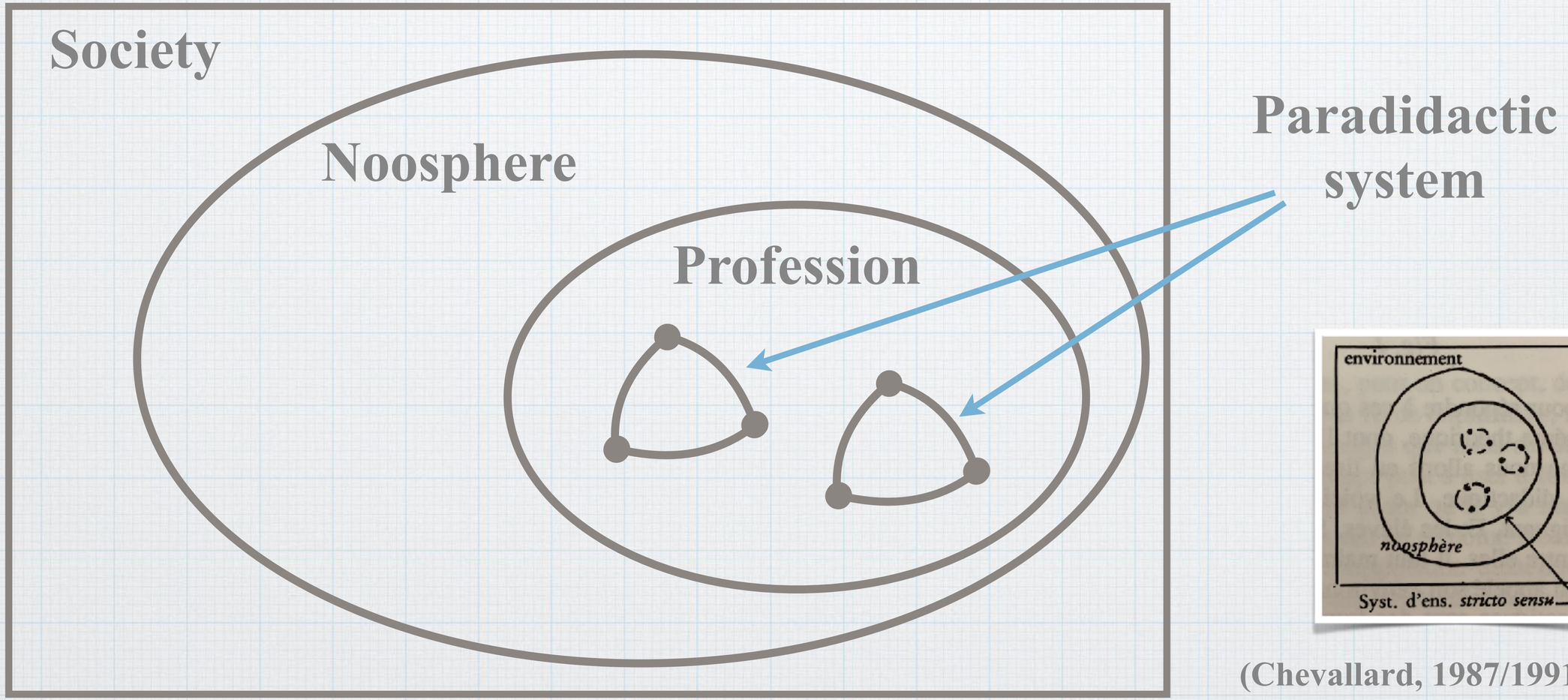
Let me introduce the notion of a *noospheric profession* denoted by $\dot{P}$, i.e., a profession involved in a noosphere.

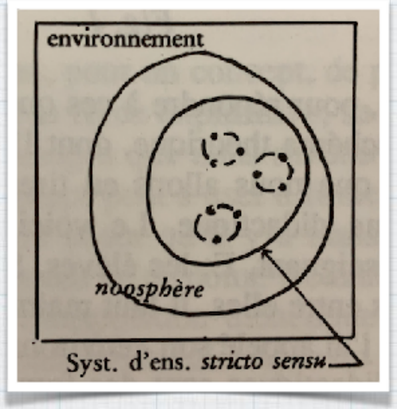
$\dot{P} := \{x \mid \mathfrak{S}(x, \dot{N}, \dot{p})\}$.

$\mathfrak{S}(x, \dot{N}, \dot{p})$ means that “x is subjected to $\dot{N}$ in a position $\dot{p}$”.

Examples of a noospheric position $\dot{p}$:
Schoolteacher, schooler, curriculum developer, textbook writer, mathematician, parent, OBOG, examinee, and so on.

Relationship between the four institutions:
After a famous image





(Chevallard, 1987/1991, p. 24)

Paradidactic analysis

3/7

Paradidactic “psychoanalysis”

In my view, paradidactic analysis is a kind of “psychoanalysis” in the *epistemological* sense of the usage of Gaston Bachelard, that is, *psychoanalysis of rationality*.

I use this term within the ATD for meaning “to analyze any instantial relation to a given object”.

☛ Please remember ATD’s *Humpty Dumpty principle*!

Paradidactic psychoanalysis aims to implicit common sense in any noosphere (profession, and paradidactic system).

Paradidactic “psychoanalysis”

There are two different types of paradidactic analysis.

* First kind: the traditional transpositive analysis.
  To study implicit, dominant epistemological models in a given noosphere.

* Second kind: analysis of noospheric didactic knowledge.
  To study implicit, dominant didactic models in a given noosphere.

  ☛ I will show you some examples from now on.

Cognitive substantialism

Substantialism is an epistemological attitude regarding any existence as an independent entity.
☛ An antonym is *relationalism*, which tries to recognize thing as a system consisting of different entities.

For example, the existence of a concept is usually substantialized as a self-contained entity in didactic (and epistemological) context.
☛ By contrast, within Vergnaud's model of concept, any concept is understood as a system of situations S, an invariants I, and a set of symbolic representations $\mathcal{S}$.

There are several symptoms of it.

* The fabrication of "illusiory" substances (e.g., astrology)

* The table complex: obsession to completing a folk taxonomy of a given fabricated category

An example from Japan (Ishoda & Katagiri, 2012, pp. 50–52)

Lists of Mathematical Thinking Types

(A) Mathematical attitudes (Mindset)

(1) Attempting to grasp one's own problems, or objectives and substance, clearly, by oneself (objectifying):
- (i) Attempting to pose questions;
- (ii) Attempting to be aware problematic;
- (iii) Attempting to realize mathematical problems from situation.

(2) Attempting to take logical reasonable actions (reasonableness):
- (i) Attempting to take actions that match the objectives;
- (ii) Attempting to establish a perspective;
- (iii) Attempting to think based on the data that can be used, previously learned items, and assumptions.

(3) Attempting to represent matters clearly and simply (clarity):
- (i) Attempting to record and communicate problems and results clearly and simply;
- (ii) Attempting to sort and organize objects when representing them.

(4) Attempting to seek better ways and ideas (sophistication):
- (i) Attempting to raise thinking from the object to the operation;
- (ii) Attempting to evaluate thinking both objectively and subjectively, by each other, for refining;
- (iii) Attempting to economize thought and effort.

(*Continued*)

(*Continued*)

(B) Mathematical thinking related to mathematical methods in general

(1) Inductive thinking
(2) Analogical thinking
(3) Deductive thinking
(4) Integrative thinking (including extensional thinking)
(5) Developmental thinking
(6) Abstract thinking (abstraction) (thinking that abstracts, concretizes, idealizes, and thinking that clarifies conditions)
(7) Thinking that simplifies (simplifying)
(8) Thinking that generalizes (generalizing)
(9) Thinking that specializes (specializing)
(10) Thinking that symbolizes (symbolizing)
(11) Thinking that represents with numbers, quantities, and figures (quantification and schematization)

(C) Mathematical thinking related to mathematical content in substance (mathematical ideas)

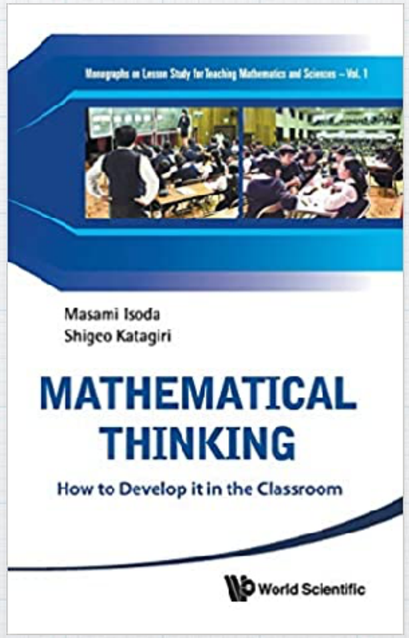
(1) Clarifying sets of objects for consideration and objects excluded from sets, and clarifying conditions for inclusion (*idea of sets*);
(2) Focusing on constituent elements (units) and their sizes and relationships (*idea of units*);
(3) Attempting to think based on the fundamental principles of representation (*idea of representation*);[1]
(4) Clarifying and extending the meaning of things and operations, and attempting to think based on this (*idea of operations*);

(*Continued*)

[1] Mathematical representations are not only limited to mathematical expressions such as mathematical sentences and formulas.

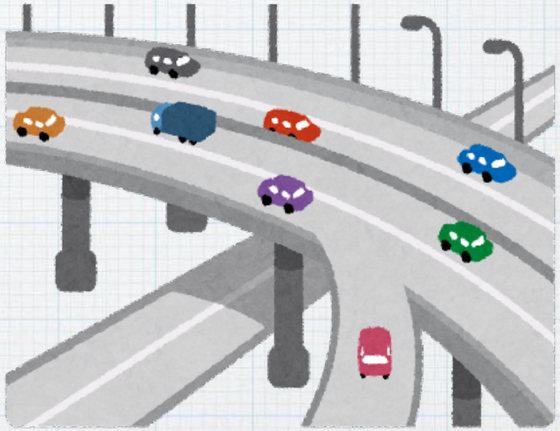
(*Continued*)

(5) Attempting to formalize operation methods (*idea of algorithms*);
(6) Attempting to grasp the big picture of objects and operations, and to use the result of this understanding (*idea of approximation*);
(7) Focusing on basic rules and properties (*idea of fundamental properties*);
(8) Attempting to focus on what is determined by one's decisions, to find and use rules of relationships between variables (*functional thinking*);
(9) Attempting to represent propositions and relationships as expressions, and to read their meaning (*idea of expressions*).



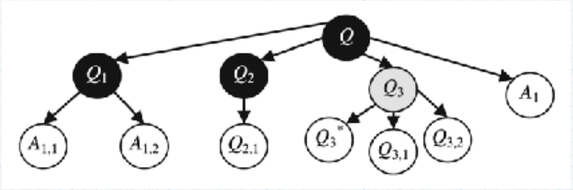
Path apriorism

* A didactic belief which regards schemas of didactic process as unchangeable formats independent of the functioning of actual didactic systems.
* Typically, various cyclic models of didactic time can be recognized as *predetermined study paths*.
* In the extreme case, whether or not *pedagogically* covering all the step of a given cyclic model becomes crucial criteria for the degree of didacticity of possibly didactic situations.



Path aposteriorism

The *Caminante* principle



(Winsløw et al. 2013, p. 271)

- Any study path is not an infrastructural constraint in advance, where students must go through, but a mass of their "study footprints" after studying.
- In the case of SRP, such footprints should be recognized as *questions* (and *answers*).
- *Caminante, son tus huellas el camino y nada más; Caminante, no hay camino, se hace camino al andar. (Antonio Machado)*

Existing psychoanalytic products

* Thematic confinement
* Pedagogical generalism
* Monumentalization of knowledge to be taught

Toward completion of the scale of didactic codeterminacy

4/7

The scale of didactic codeterminacy levels

Humankind ↔ Civilizations ↔ Societies

↓↑

Schools ↔ Pedagogies ↔ Didactic systems

The scale of didactic codeterminacy levels

Protodidactic (or anthropological) reality

Humankind ↔ Civilizations ↔ Societies

↓↑

(narrowly) Didactic reality

Schools ↔ Pedagogies ↔ Didactic systems

An “explicated” scale

Protodidactic reality

↓↑

An “ecotone” reality between the two realities

The habitat of noospheres, professions and paradidactic systems

↓↑

Didactic reality

An “explicated” scale

Humankind ↔ Civilizations ↔ Societies

↓↑

Noospheres ↔ Professions ↔ Paradidactic systems

↓↑

Schools ↔ Pedagogies ↔ Didactic systems

An “explicated” scale

Protodidactic (or anthropological) reality

Humankind ↔ Civilizations ↔ Societies

↓↑

Paradidactic reality

Noospheres ↔ Professions ↔ Paradidactic systems

↓↑

(narrowly) Didactic reality

Schools ↔ Pedagogies ↔ Didactic systems

A typical habitat of didactic paradigms

Humankind ↔ Civilizations ↔ Societies

↓↑

Didactic paradigms ↔ Noospheres ↔
↔ Professions ↔ Paradidactic systems

↓↑

Schools ↔ Pedagogies ↔ Didactic systems

They are "transcendental" didactic frameworks integrating various noospheres.

Didactic paradigms and stakes

Humankind ↔ Civilizations ↔ Societies

↓↑

[*Visiting works*] ↔ Noospheres ↔
↔ Professions ↔ Paradidactic systems

↓↑

Schools ↔ Pedagogies ↔ DS of Monuments

Didactic paradigms and stakes

Humankind ↔ Civilizations ↔ Societies

↓↑

[*Visiting works*] ↔ Noospheres ↔
↔ Professions ↔ Paradidactic systems

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔
↔ Sectors ↔ Themes ↔ Subjects

Where is the level of questions?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*Visiting works*] ↔ Noospheres ↔
↔ Professions ↔ Paradidactic systems

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔
↔ Sectors ↔ Themes ↔ Subjects

Where is the level of questions?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*VW*] ↔ Noospheres ↔ [*Traditional curricular project*] ↔
↔ Professions ↔ Paradidactic systems ↔ [*Exercises to be done*]

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔
↔ Sectors ↔ Themes ↔ Subjects ↔ Questions

Where is the level of questions?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*VW*] ↔ Noospheres ↔ Professions ↔ [*TDS*] ↔
↔ Paradidactic systems ↔ [*Fundamental situations*]

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔
↔ Sectors ↔ Themes ↔ Questions ↔ Subjects

Where is the level of questions?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*VW*] ↔ Noospheres ↔ Professions ↔ [*ATD*] ↔
↔ Paradidactic systems ↔ [*SRA*]

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ Domains ↔
↔ Sectors ↔ Themes ↔ Questions ↔ Subjects

Under the paradigm of questioning the world?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*Questioning the world*] ↔ Noospheres ↔ Professions ↔
↔ [*ATD*] ↔ Paradidactic systems ↔ [*Unfinalized SRP*]

↓↑

Schools ↔ Pedagogies ↔ Questions ↔
Disciplines ↔ Domains ↔ Sectors ↔ Themes ↔ Subjects

Under the paradigm of questioning the world?

Humankind ↔ Civilizations ↔ Societies

↓↑

[*QW*] ↔ Noospheres ↔ Professions ↔ [*ATD*] ↔
↔ Paradidactic systems ↔ [*Finalized SRP*]

↓↑

Schools ↔ Pedagogies ↔ Disciplines ↔ (Questions) ↔ Domains ↔
↔ (Questions) ↔ Sectors ↔ Themes ↔ Subjects

The complexity of teacher education

5/7

Some questions about TE

1. What kind of *institutional transposition* of knowledge happens for teacher education?
2. What kinds of didactic systems are set in teacher education?
3. Why are such different types of didactic systems involved in teacher education?

Normal school

Let me call here any institution for teacher education a *normal school* in contrast with the entity of *school in a narrow sense*.

* Specialized college for TE (e.g., Danish “university college”)
* A department of didactics in university
* Of course, “normal school” itself (in Japan, it is a historical entity)

Any normal school is a didactic subinstitution of the teaching profession.

Ideal types of transpositions

There are some kinds of transposition of knowledge from an institution to another.

The most general notion is *institutional transposition*, which could have several subtypes as follows:

* "Didactic" transposition: from a production institution to a "school" in a narrow meaning
* *Archididactic transposition*: from a production institution to an application institution (e.g., from a mathematicians institution to a physicists institution)
* *Initiating transposition*: from a production institution to its didactic subinstitution (e.g., from a legal profession to a law school)
  ☛ The *paradigm of granting membership*: real experience is the best teacher!

In the case of normal school

* The archididactic transposition of disciplinary knowledge (e.g., mathematics, language, and philosophy), which is *exogenetic* for the teaching profession.

* The initiating transposition of schoolteachers' *homemade* didactic knowledge, which is *endogenetic* for the teaching profession.

  ☛ That can be called *paradidactic transposition*.

Didactic systems in normal school

* **Archididactic systems: $S(\dot{X}, \check{Y}, \lozenge)$.**
  $\dot{X}$: prospective teachers.
  $\check{Y}$: professionals outside the teaching profession.
  $\lozenge$: Knowledge at stake for which $\check{Y}$ is professionalized.

  ☛ **Archididactic stake**

* **Paradidactic systems: $S(\dot{X}, \dot{Y}, S(X, Y, \heartsuit))$**
  $\dot{X}$: prospective teachers
  $\dot{Y}$: e.g., pedagogues and "charismatic" schoolteachers
  $S(X, Y, \heartsuit)$: prepared clinically and/or fictively

The functionality of the two kind of didactic systems

The functioning of didactic systems : $S(X, Y, ♥) \rightarrow R(X, ♥)$.

☛ $S(\dot{X}, \dot{Y}, S(X, Y, ♥)) \rightarrow R(\dot{X}, S(X, Y, ♥))$

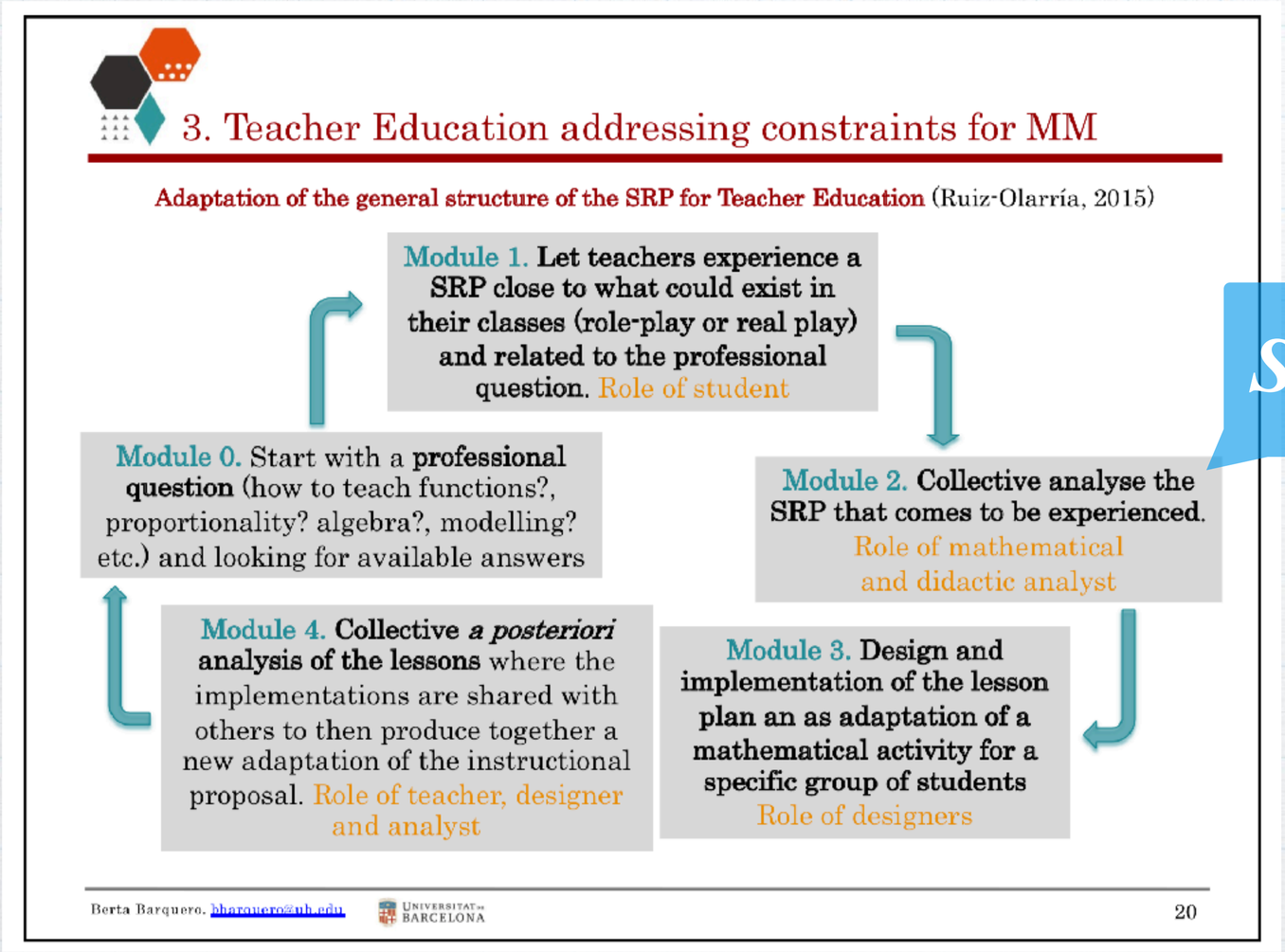
The developed schema: $[S(X, Y, ♥) \rightarrow M] \rightarrow R(X, ♥)$.

☛ $[S(\dot{X}, \dot{Y}, S(X, Y, ♥) \rightarrow \{\Diamond_1, \Diamond_2, .., \Diamond_i\})] \rightarrow R(\dot{X}, S(X, Y, ♥))$

Archididactic systems $S(\dot{X}, \check{Y}, \Diamond)$ **are for giving the elements** $\Diamond_1, \Diamond_2, ..., \Diamond_i$ **of** ***paradidactic milieus*****.**

Paradidactic systems $S(\dot{X}, \dot{Y}, S(X, Y, ♥))$ **are for constructing the homemade didactic knowledge in the teaching profession.**

SRP-TE: $S(\dot{X}, \dot{Y}, S(X, Y, q))$



$S(\dot{X}, \dot{Y}, S(\dot{X}, \dot{Y}, q)$

(Barquero, 2022, presentation in an online seminar)

Normal noosphere

Any noosphere is a noosphere of “some school”.

The team “noosphere” in the ATD usually means the noosphere of “school” in a narrow sense.

We can consider about the “noosphere of normal school”, which can be named a *normal noosphere*.

* Teacher trainers at the moment of the writing of their syllabi.
* The committee for the management of normal school.
* Authors of journal papers and/or textbooks for the teaching profession. etc.

The complex ecosystem around TE

Protodidactic reality

↓↑

Normal school paradidactic reality

↓↑

The niche of TE

"School" paradidactic reality ↔ Normal school didactic reality

↓↑

The niche of TE

"School" didactic reality

Reflexive didactics

6/7

Reference epistemological model

ATD-didacticians are supposed to construct *reference epistemological models* (REM) for studying didactic reality.

Roughly speaking, any RME is a model $\mathfrak{M}$ of a certain work w—its kind and size do not matter—within the didacticians' institution $\dot{D}$.

Generalizing the notion of REM

Let me express a REM by $\mathfrak{M}_{\dot{D}}(w)$.

We can suppose other possible "reference" models of w.

* REM of production institution $\check{P}$: $\mathfrak{M}_{\check{P}}(w)$.

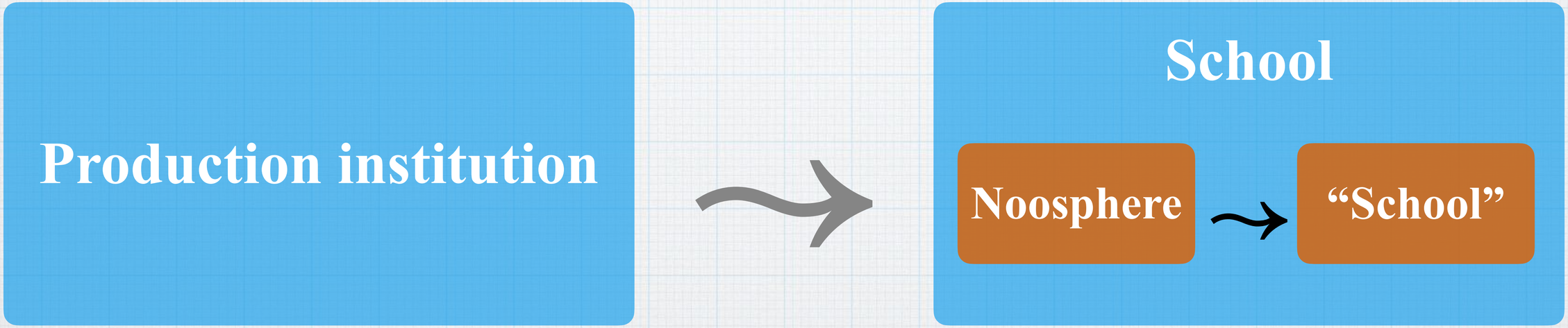
* REM of school Σ: $\mathfrak{M}_{\Sigma}(w)$.

Such generalization of RME leads us to reflect on the process of didactic transposition.

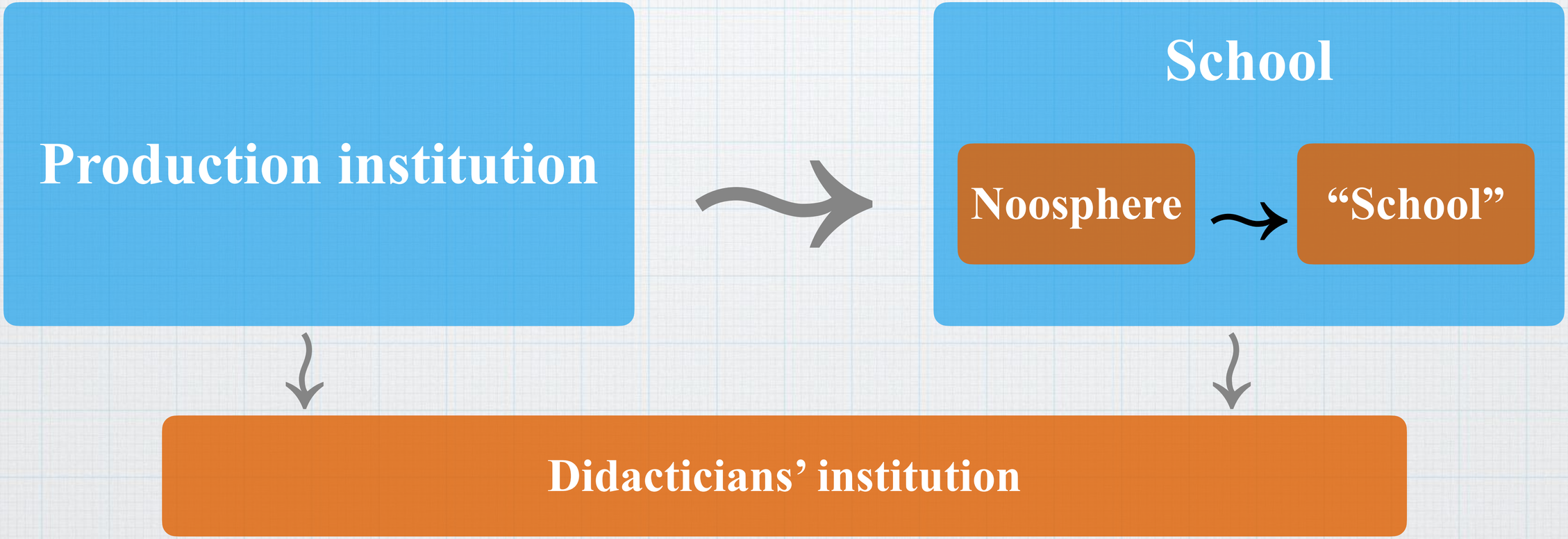
Some degrees of approximation of didactic transposition process

Production institution → School

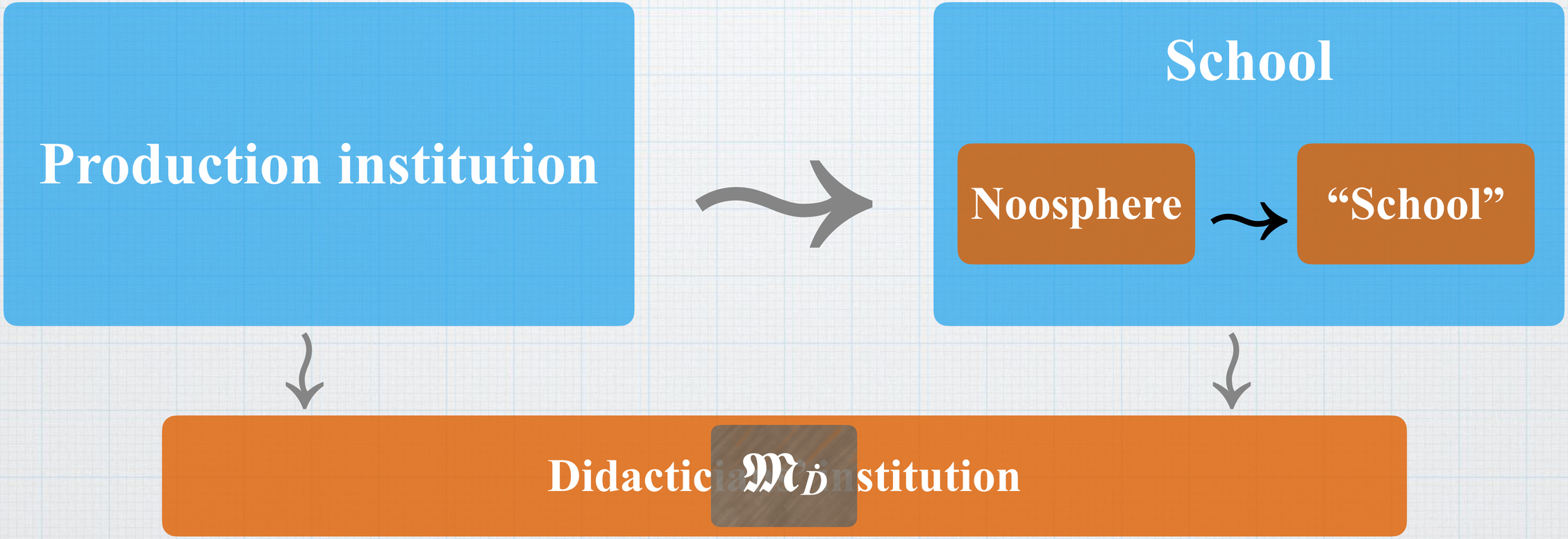
Some degrees of approximation of didactic transposition process



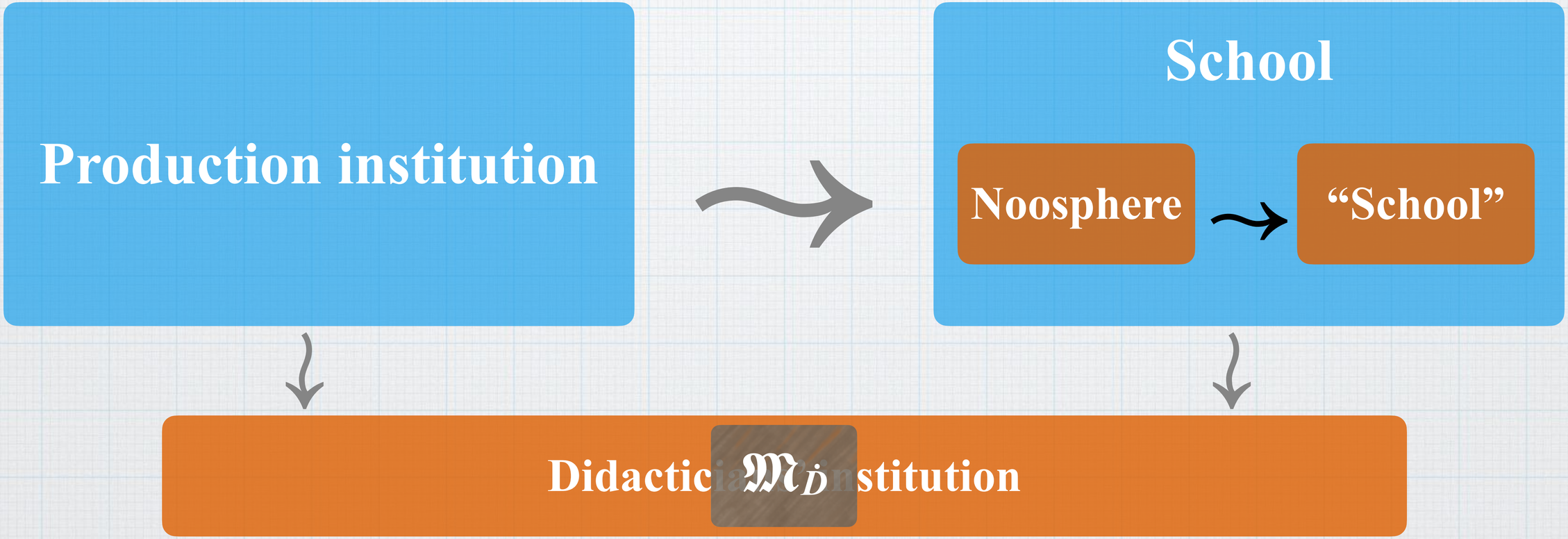
Some degrees of approximation of didactic transposition process



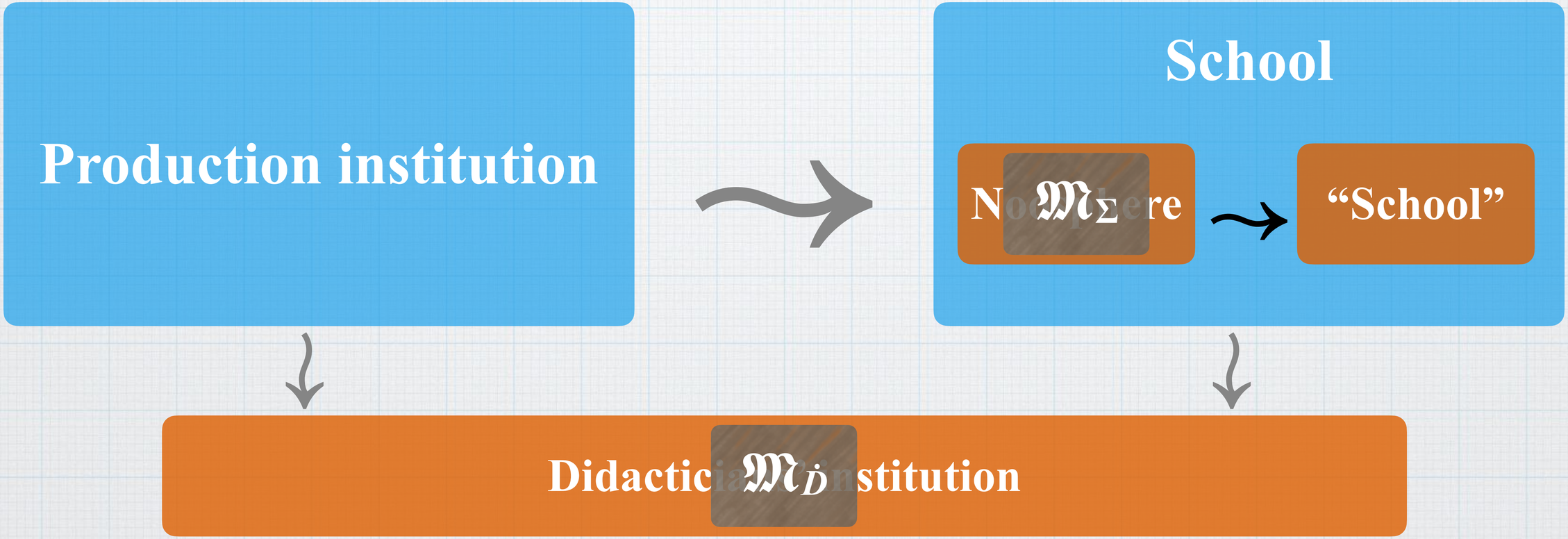
Where does $\mathfrak{M}_{\dot{D}}$ live?



Where does $\mathfrak{M}_\Sigma$ live?



Where does $\mathfrak{M}_\Sigma$ live?



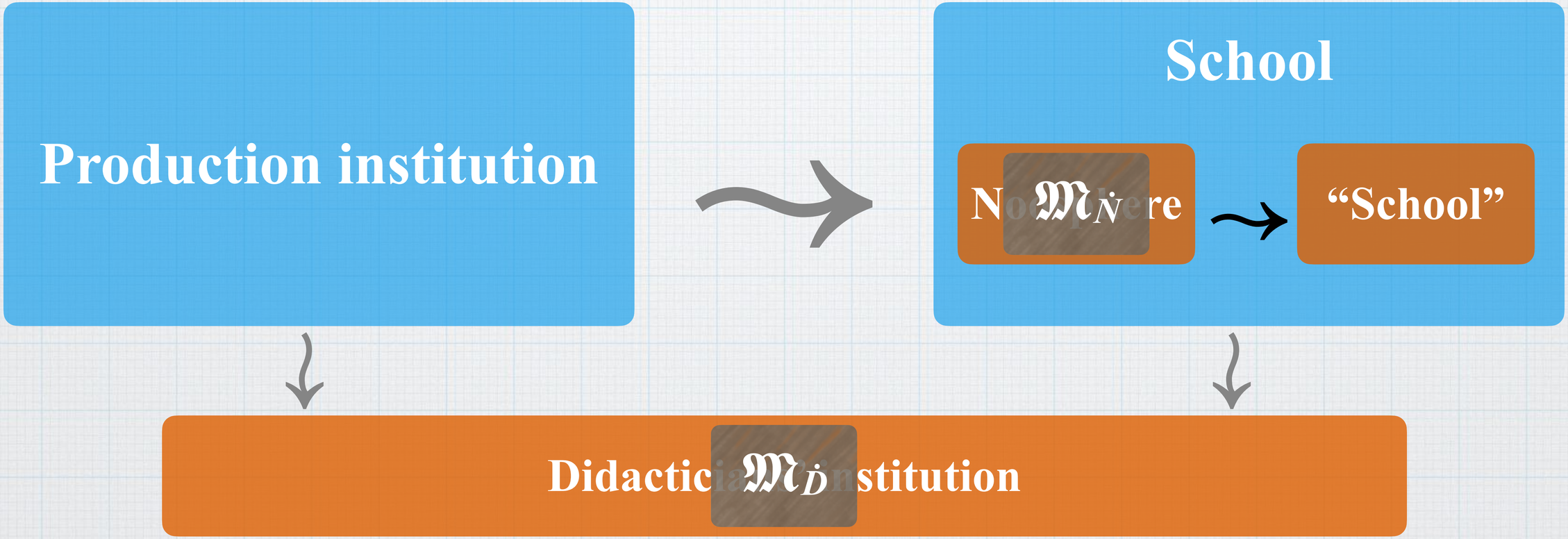
The “transcendentality” of REM

A reference “epistemological” model is any kind of “knowledge about knowledge”, “theory about theory”, or “model of model”.

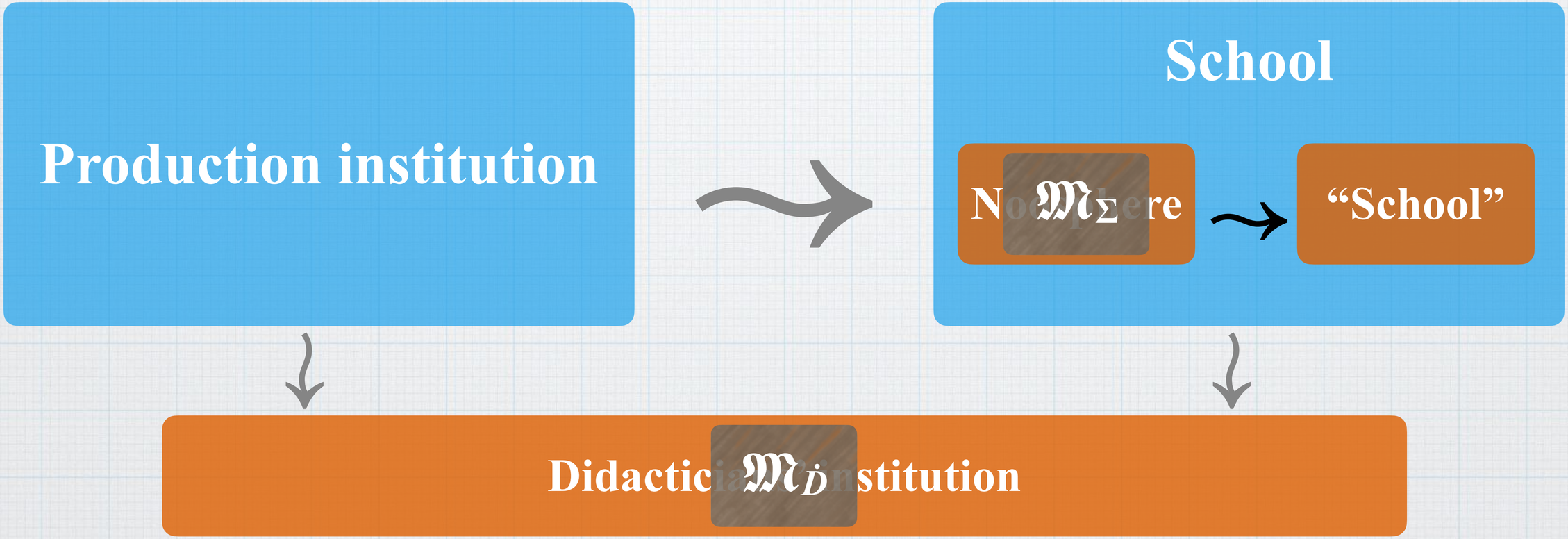
I call such metatheoretical nature the transcendentality.

The transcendentality means that “epistemological” models are produced by the functioning of paradidactic systems $S(\dot{X}, \dot{Y}, S(X, Y, \heartsuit))$.

$$\mathfrak{M}_{\Sigma} = \mathfrak{M}_{\dot{N}}$$



Where does $\mathfrak{M}_{\check{P}}$ live in?



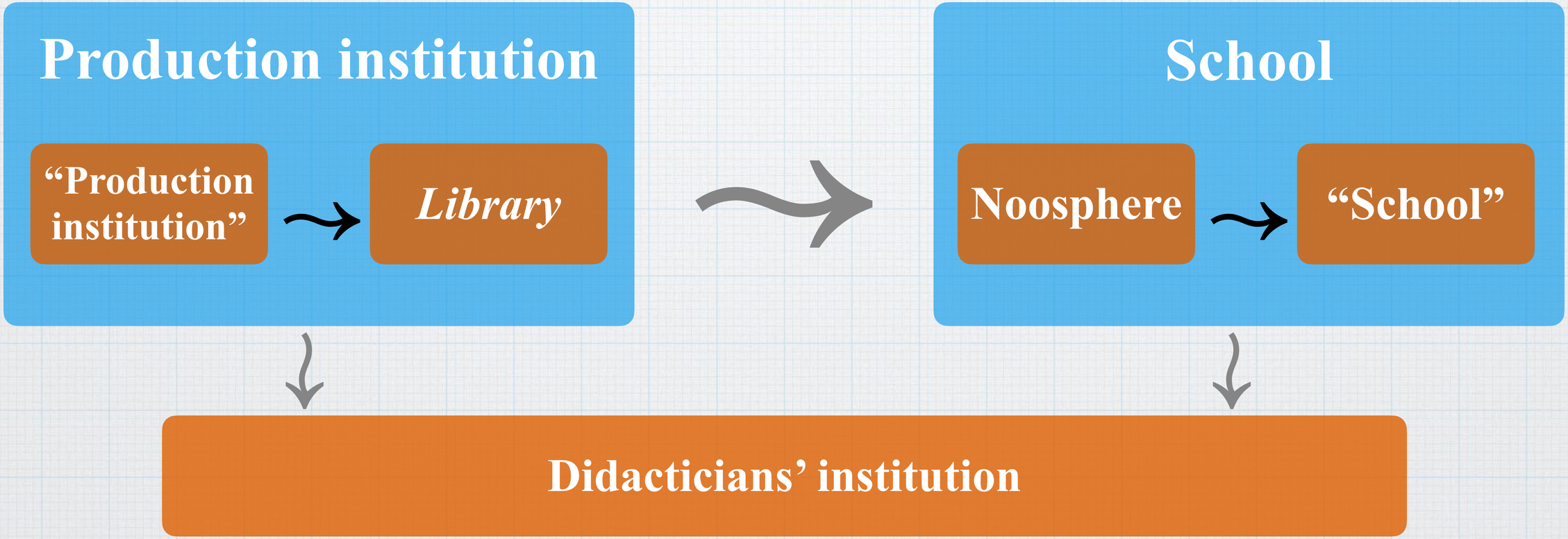
The institution of “library”

The institution who accomplishes the following tasks for making “noble” knowledge produced more *understandable*.

* Gathering works of knowledge
* Selecting important parts out of gathered works
* Reorganizing selected works

Examples: Epistemologists, historians, editorial board of journals, and writers of “technical books” (e.g., the GTM series of Springer and the Bourbaki’s treatises).

Some degrees of approximation of didactic transposition process



An application of the notion of library

Mathematics education around the world has many things to be taught which we find it difficult to explain them in the framework of didactic transposition.

* Metacognitive skill, modeling process, competency, inquiry, and so on and so far.

They never come directly from (the narrowly) production institution of mathematics to noosphere.

Where do they come from?—From libraries!

Revisiting the notion of noosphere

* The noosphere $\dot{N}$, the library $Ł$, and, the didacticians' institution $\dot{D}$ have in common their transcendental positioning in paradidactic systems $S(\dot{X}, \dot{Y}, S(X, Y, \heartsuit))$, where possibly $\dot{X}, \dot{Y} \in \dot{N}$, $Ł$, or $\dot{D}$.

* Please remind that $\dot{N} := \{x \in \check{S}_\Sigma \mid R(x, \Sigma) \neq \emptyset\}$.

* In the broadest meaning of the word "noosphere", *didacticians also are noospherians*.

* This means that the paradidactic theory can be useful for epistemological vigilance for ourselves.

Final remarks:
on the professionalization of the teaching profession

7/7

The teaching profession as a semiprofession

* The teaching profession tends not to be regarded as a full-fledged profession, e.g., lawyer and physician.

* How do we ATD-didacticians empower the teaching profession?
  My personal answer: to give rich didactic and epistemological resources from the ATD. (e.g., Q-A map)

Thank you very much

for your following the *principle of charity*.